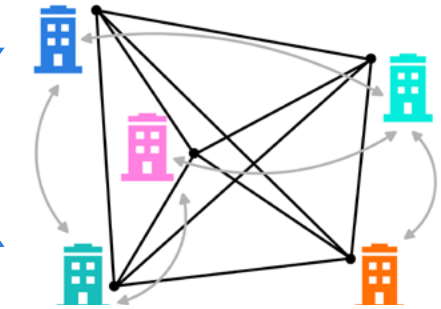
D-WAVE PERFORMANCE STUDY:
FACILITY PLACEMENT

OPTIMIZING THE QUADRATIC ASSIGNMENT PROBLEM

The Quadratic Assignment Problem (QAP) is to select locations that minimize the total flow cost between facilities.

The optimization tasks for this problem include minimizing transportation costs, reducing travel distances, and optimizing resource allocation. These tasks are commonly found in industries where efficient layout planning and logistics are essential, such as manufacturing, supply chain management, and facility layout design.

In QAP, we are given a set of n facilities and n locations. Between each pair of facilities, there is a "flow" (for example, the amount of supplies transported from one facility to the other). The goal is to place the facilities (one per location) so that the sum of the flows times the distances is minimized. The objective function is nonlinear in the problem variables, which makes it difficult to formulate for MILP and LP solvers.



QUADRATIC ASSIGNMENT PROBLEM

On QAP problems from the Taillard benchmarking library with 150 seconds of runtime, the gap for solutions found by D-Wave's Hybrid Nonlinear-Program solver, or NL solver, beats or ties all "b" and "c" instances. For the "a" instances, the only solver to beat the NL solver is D-Wave's CQM solver.

PROBLEM INSTANCES

The instances benchmarked for this paper are the Taillard instances [1] and [2]. These 26 problems contain the distance between each pair of locations and the flow between each pair of facilities, with problem sizes ranging from 12 to 256 locations (the number of locations is equal to the number of facilities). These instances, categorized as "a", "b", and "c", are as follows:

- "a" instances are from [1] and are uniformly generated.
- "b" instances are from [2] and are randomly generated, with distance matrices containing Euclidean distances between pairs of n randomly generated points in the plane.
- "c" instances are from [2] and have binary flow matrices with a block of 1s in the upper left corner and are inspired by the problem of printing greys of a given density.

MATHEMATICAL MODELING

This section discusses the various mathematical models that were used in this study.

MIXED INTEGER NONLINEAR PROGRAMMING (MINLP) MODEL

For MINLP solvers, we use binary variables x_{ij} to indicate whether facility *i* is placed in location *j*. Thus the number of variables is quadratic in the number of locations. If F_{ik} is the flow from facility *i* to facility *k*, and D_{jl} is the distance between location *j* and location *l*, then we aim to minimize the objective function:

$$\sum_{i,j,k,l} F_{ik} D_{jl} x_{ij} x_{kl}$$

This sum is over all pairs of facilities *i*, *j* and all pairs of locations *k*, *l*. We also encode constraints to ensure that there is only one facility placed in each location and only one location per facility. The number of constraints is twice the number of locations.

This formulation is used by D-Wave's hybrid constrained quadratic model (CQM) solver, COIN-OR's Ipopt (Interior Point Optimizer), and OR-Tools CP-SAT solver.

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SCIPY QUADRATIC ASSIGNMENT FUNCTION

SciPy provides a quadratic-assignment function that includes its 2-opt method, a local search algorithm that searches by swapping facility placements.

NONLINEAR MODEL

D-Wave's hybrid nonlinear-program (NL) solver can efficiently encode a QAP problem by taking advantage of a list variable that encodes an ordering of the facility placements. This variable eliminates the need for both the quadratic number of binary variables representing facility placements and the constraints preventing multiple facilities per location (and vice versa) required by the MINLP formulation. The arrays of flows and distances are converted to constant variables from which the objective value is computed.

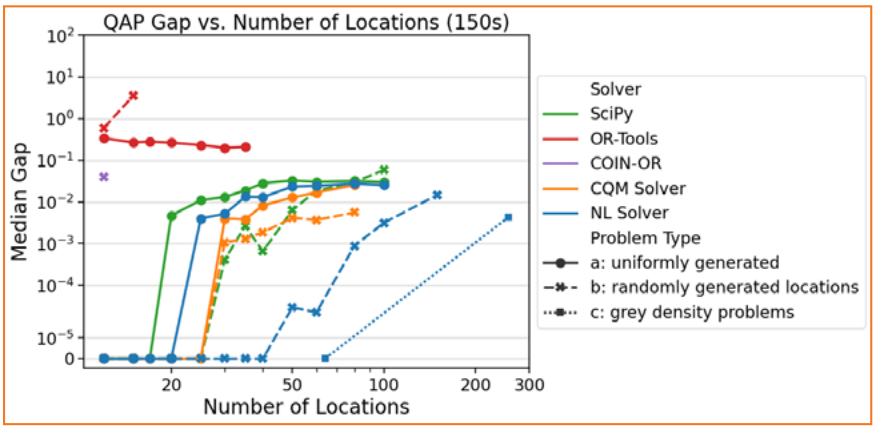
```
model = Model()

# Add the flow and distance matrices
F = model.constant(flows)
D = model.constant(distances)

# Create a list variable to encode facility placement
n = distances.shape[0]
x = model.list(n)

# Minimize the sum of the flows times the distances
model.minimize((F * D[x, :][:, x]).sum())
model.lock()
```

RESULTS

All problems were run with a time limit of 150 seconds. Results are reported as optimality gaps (energy divided by the best known solution minus 1) when feasible. Infeasible solutions correspond to infinite gaps in the median, and if the median is infeasible, the data point is not shown in the plot. In order to impose time limits on COIN-OR's solver, presolve techniques are turned off. Solvers that terminate faster than the time limit are rerun for the full allocated time and the minimum objective value found is saved.

D-Wave's NL solver and CQM solver benchmarks ran on the Leap™ quantum cloud service. COIN-OR, OR-Tools, and SciPy were run on an Intel Core i9-7900X CPU @ 3.30GHz processor with 16GB RAM. The benchmarks for OR-Tools were run with eight threads, and the remaining were run with a single thread.

The following graph shows results on the Taillard QAP instances with a time limit of 150 seconds.



The complete study contains more time limits. For all "b" and "c" instances, the NL solver beats or ties all other solvers. For all "a" instances, the only solver to beat the NL solver is D-Wave's CQM solver. The NL solver is the only solver able to obtain a solution for the problems of size 150 and 256 for all time limits tested.

Full experimental data for feasible solutions can be downloaded as a file here.

REFERENCES

[1] Taillard, E. D. "Robust taboo search for the quadratic assignment problem." Parallel Computing, Vol. 17, nos. 4-5 (July 1991): 443-455. https://doi.org/10.1016/S0167-8191(05)80147-4

[2] Taillard, E. D. "Comparison of iterative searches for the quadratic assignment problem." Location Science, Vol. 3, no. 2 (August 1995): 87-105. https://doi.org/10.1016/0966-8349(95)00008-6