DENSO
Crafting the Core

Partition of Large Optimization Problems with One-Hot Constraint

Shuntaro Okada[1,2], Masayuki Ohzeki[2] and Shinichiro Taguchi[1]

[1]DENSO CORPORATION
[2]Tohoku University

DENSO × QUANTUM 3

TOHOKU UNIVERSITY

Take-home messages

Motivation:
Solving large optimization problems
with the one-hot constraint efficiently.

Message1:
For difficult optimization problems with frustrations,
the proposed methods are effective in improving solutions.

Message2:
One of the proposed methods releases us from adjusting λ,
which controls the strength of the one-hot constraint.

Agenda

1.Optimization of large problems using D-Wave

2.Proposed methods

3.Assessment of solution accuracy

4.Discussion on the results

5.Summary

1.

Optimization of large problems using D-Wave

Limitations of the current D-Wave machine

■Ising model of D-Wave machine

①Number of qubits

$$\mathcal{H} = \sum_{(i,j)\in Chimera}^{N_q} J_{ij}\sigma_i\sigma_j + \sum_{i=1}^{N_q} h_i\sigma_i$$

②Restricted to Chimra graph

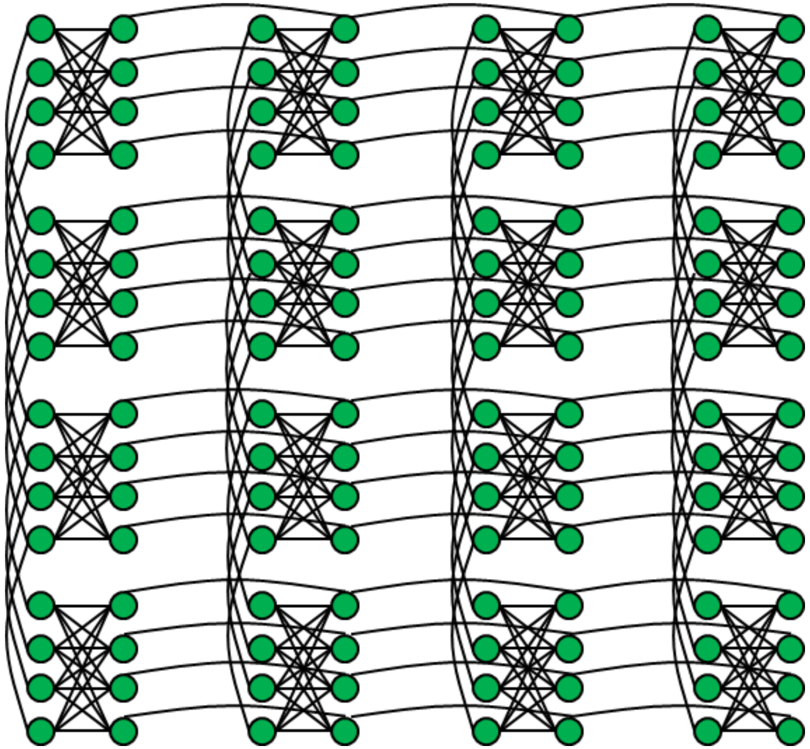
■Practical optimization problem

①Large number of variables

$$\mathcal{H} = \sum_{i<j}^{N_p} J_{ij}x_ix_j + \sum_{i=1}^{N_p} h_ix_i$$

②Between arbitrary variables

■Structure of Chimera graph

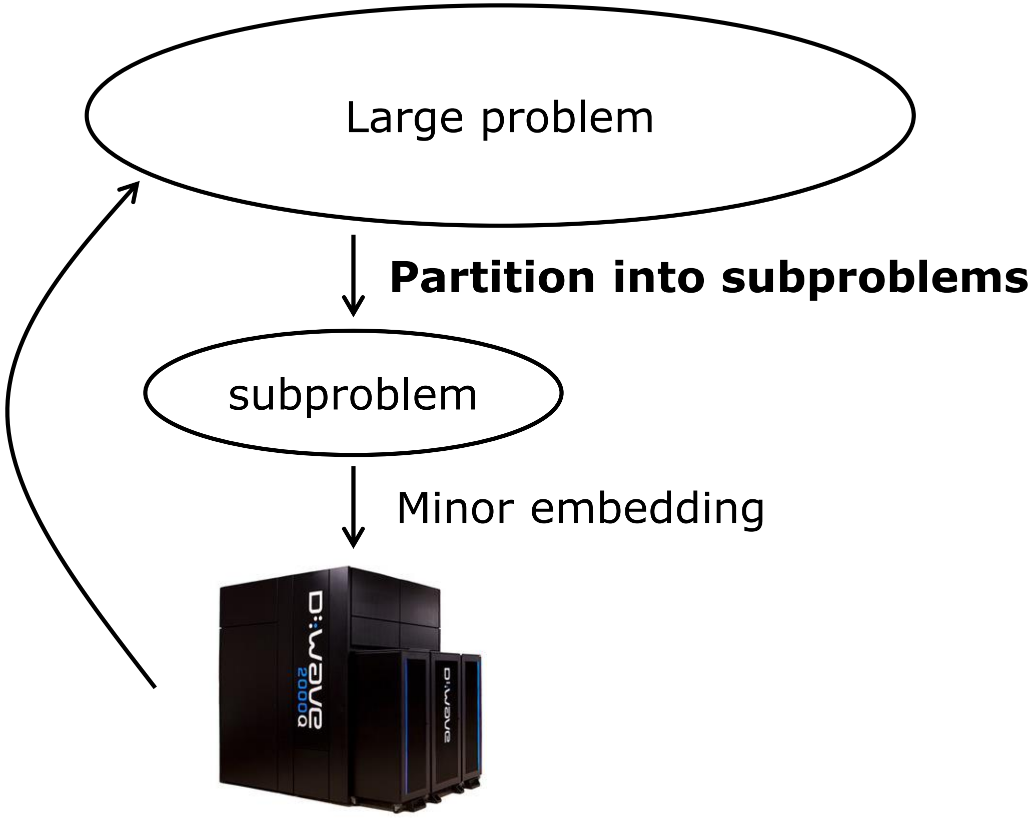


Partitioning and embedding are required to solve practical optimization problems.

In this talk, we focus on the partition of large optimization problems.

Conventional tool: qbsolv

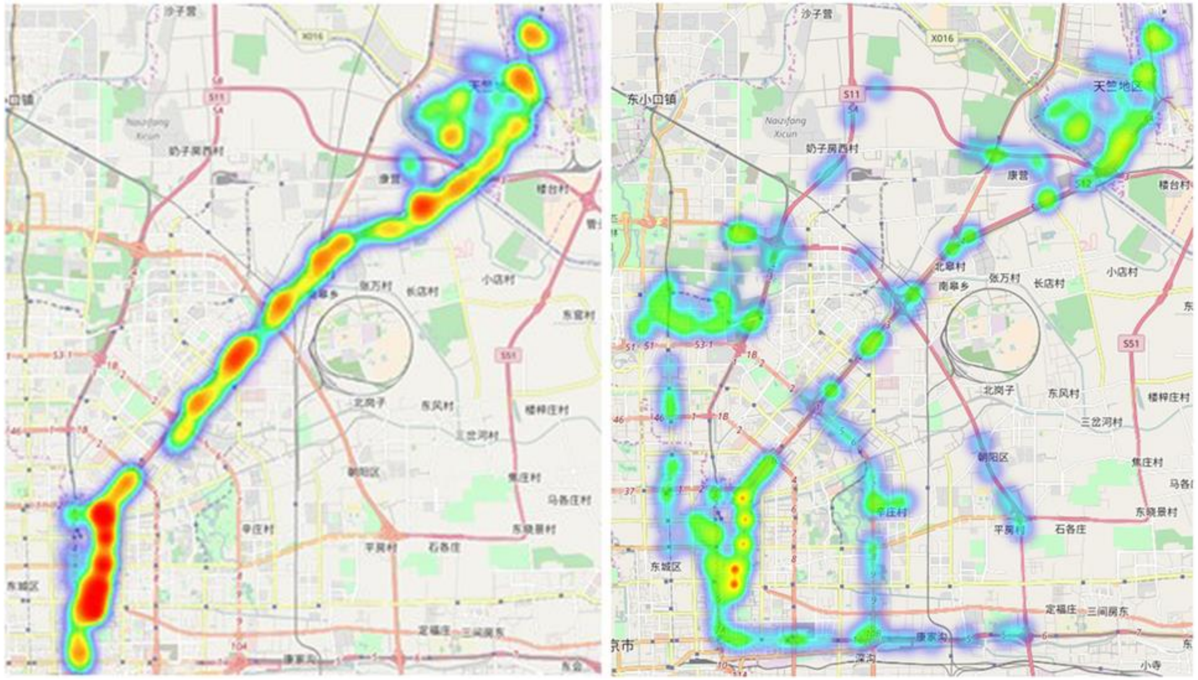
■Optimization process of qbsolv



We propose efficient partitioning of large problems with the one-hot constraint.

■Example of a problem with one-hot constraint

< Traffic flow optimization by VW >



F. Neukart, et. al., Front. ICT **4**, 29 (2017)

Select one route from three options for each taxi to minimize traffic congestion

We propose efficient partitions for the problems with one-hot constraint.

2.

Proposed methods

One-hot representation

<Original cost function>

$$\mathcal{H}_0 = \sum_{i<j} J_{ij}\delta(S_i, S_j)$$

$$S_i \in (1, 2, ..., Q)$$

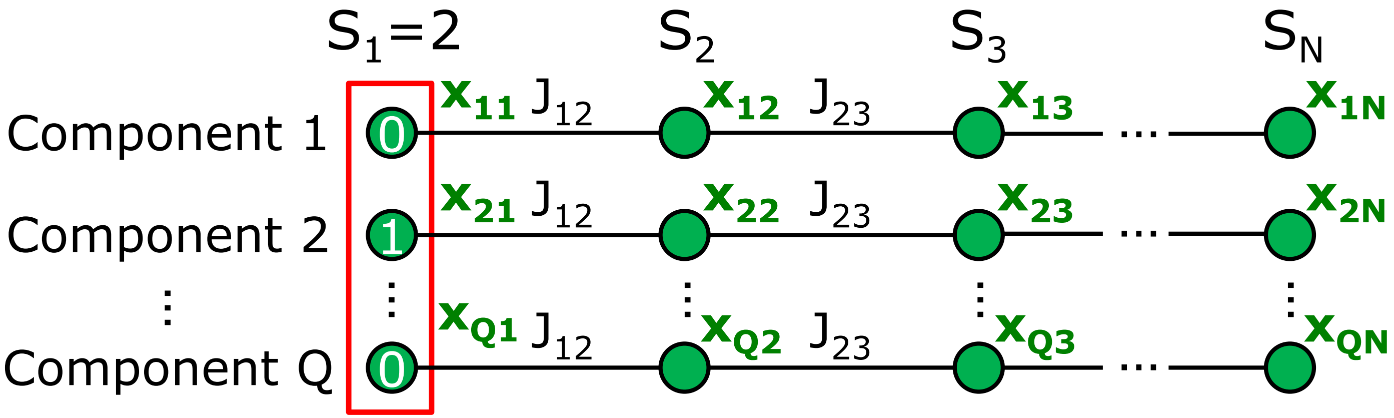
variable with Q components

<Cost function with one-hot constraint>

$$\mathcal{H}_0 = \sum_{i<j} J_{ij} \sum_{q=1}^{Q} x_{qi}x_{qj} + \lambda \sum_{i=1}^{N} \left(\sum_{q=1}^{Q} x_{qi} - 1 \right)^2$$

one-hot constraint

$$x_{qi} \in (0, 1)$$



Only Q of 2^Q states satisfy the one-hot constraint for each S_i.

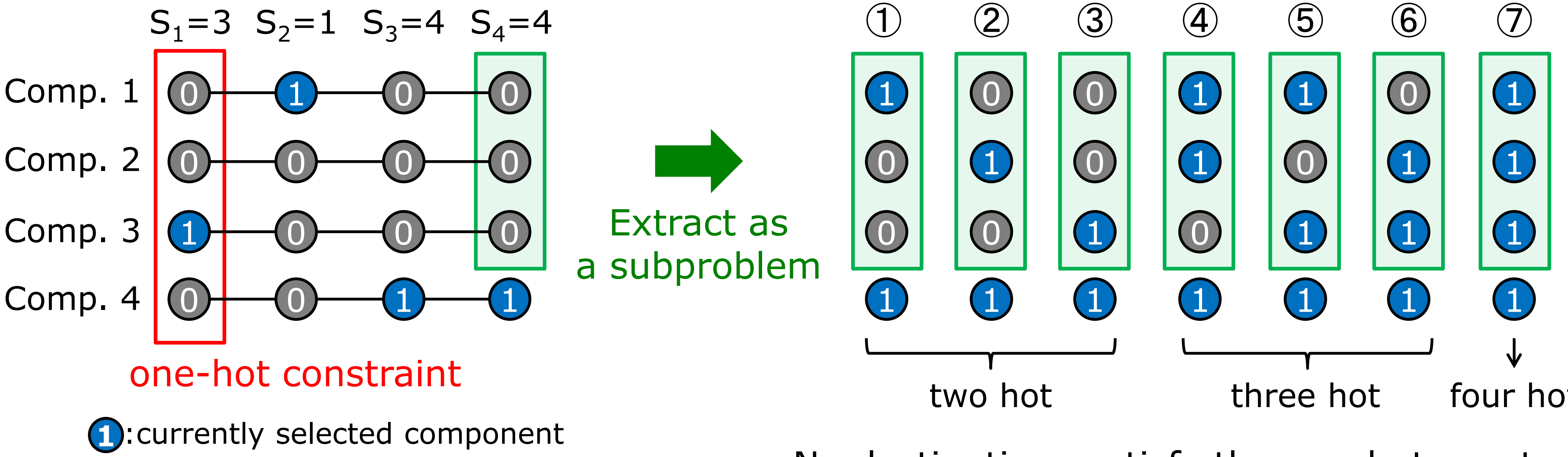
A fraction of states satisfy the one-hot constraint.

Simple example of bad partition

We should pay attention whether states satisfying the constraint are included or not.

< Current solution >

< Candidates of transition destinations>

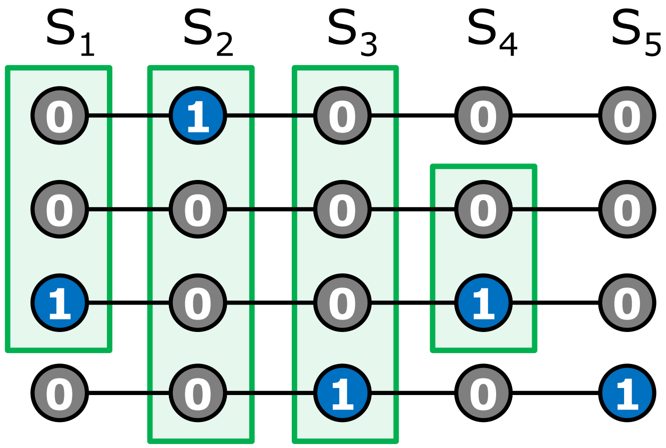
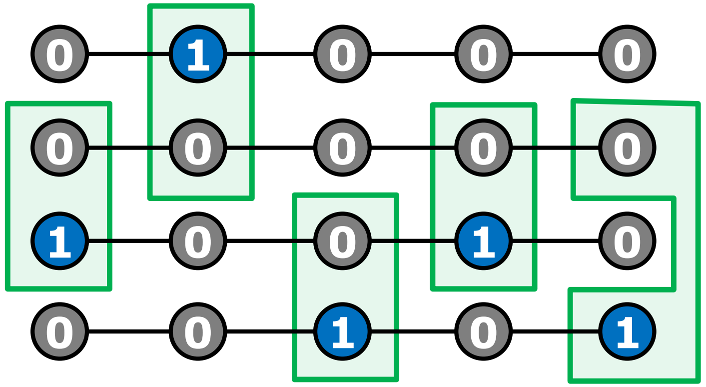
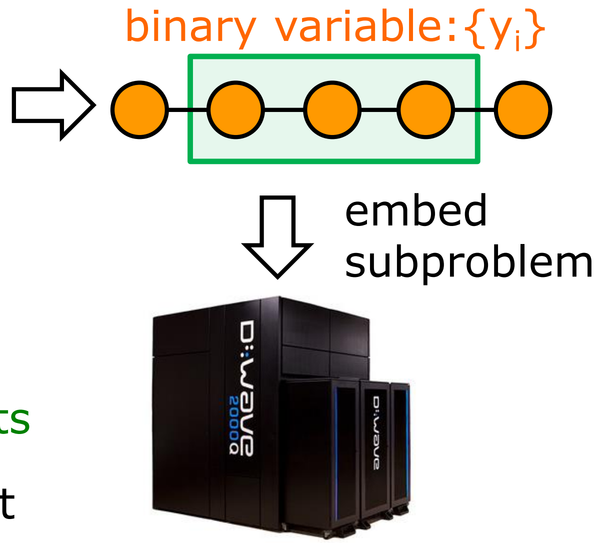


No destinations satisfy the one-hot constraint.

Better solutions cannot be searched by the optimization of the subproblem.

Proposed methods

Two methods to extract subproblems including states that satisfy the constraint.

	Multivalued partition	Binary partition
Summary	Extract subproblems that contain current comp. for each variable  subproblem (more than two components) 1 :currently selected component	Extract a binary subproblem. Select exactly one component in addition to the current comp.  subproblem with two components 1 :currently selected component Make binary subproblme; "stay or transit" 
Pros	･Destinations satisfying the one-hot constraint exist for all variables.	･All states satisfy the constraint, and λ disappears. ･Large number of variables can be embedded.
Cons	･Not all states satisfy the constraint.	･Only two components are considered at one time.

3.

Assessment of solution accuracy

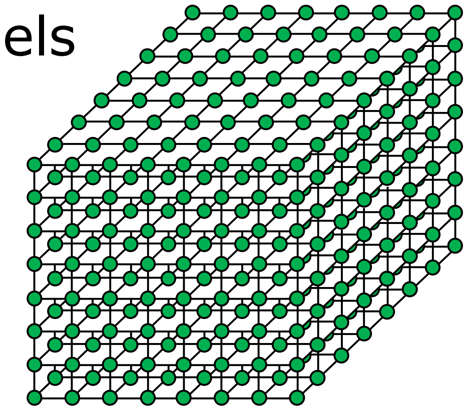
Problem settings 1/2

■Cost function

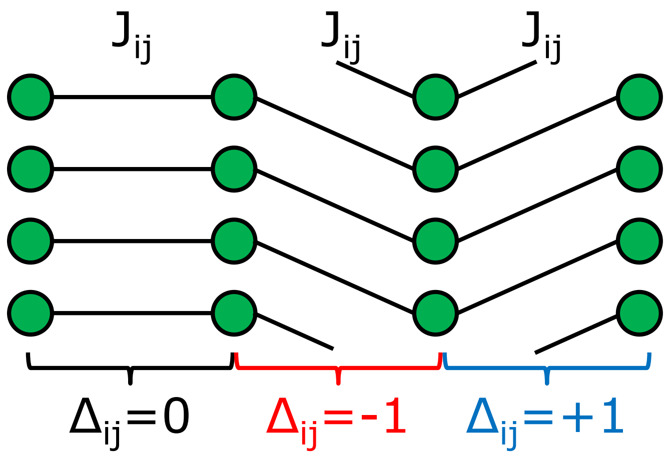
3D-4 components Potts models
with 10x10x10 variables

$$\mathcal{H}_0 = -\sum_{<ij>} J_{ij}\delta_{S_i, S_j+\Delta_{ij}}$$

$$S_i \in (1,2,3,4)$$



■Problem graph



■Parameters

model	Jij	Δij	
Ferromagnetic model	-1	0	Easy problem
Anti-ferromagnetic model	+1	0	Easy problem
Potts glass model	+1(50%) or -1(50%)	0	Hard problem
Potts gauge glass model	-1	0(50%) or +1(25%) or -1(25%)	Hard problem

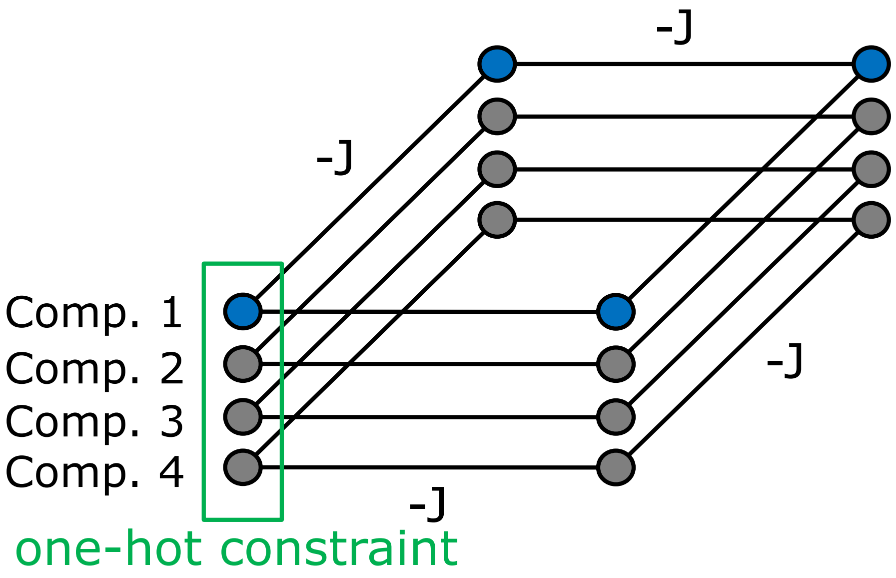
We evaluate solution accuracy for the several Potts models.

Problem settings 2/2

Simple examples of a ground state of the Potts models.

■Ferromagnetic Potts model

Interactions between same components exist.

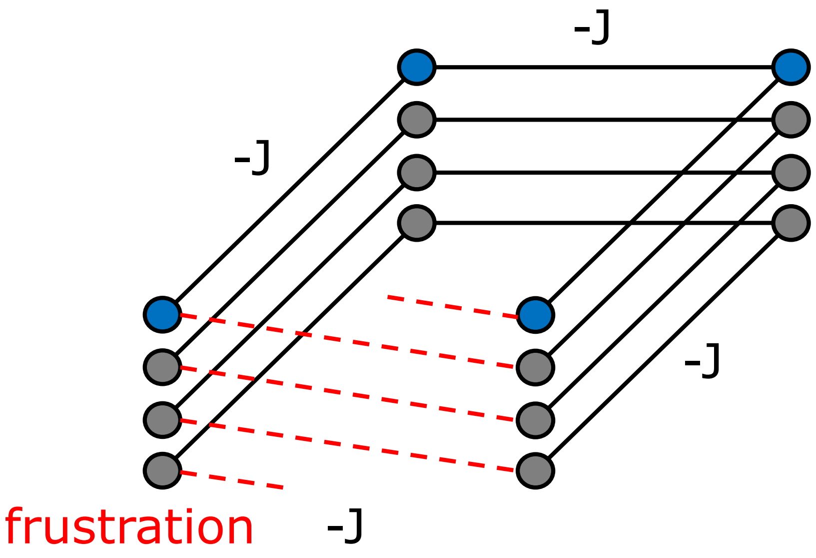


:selected component

All interactions can be satisfied;
trivial ground state

■Potts gauge glass model

Interactions between different components exist.

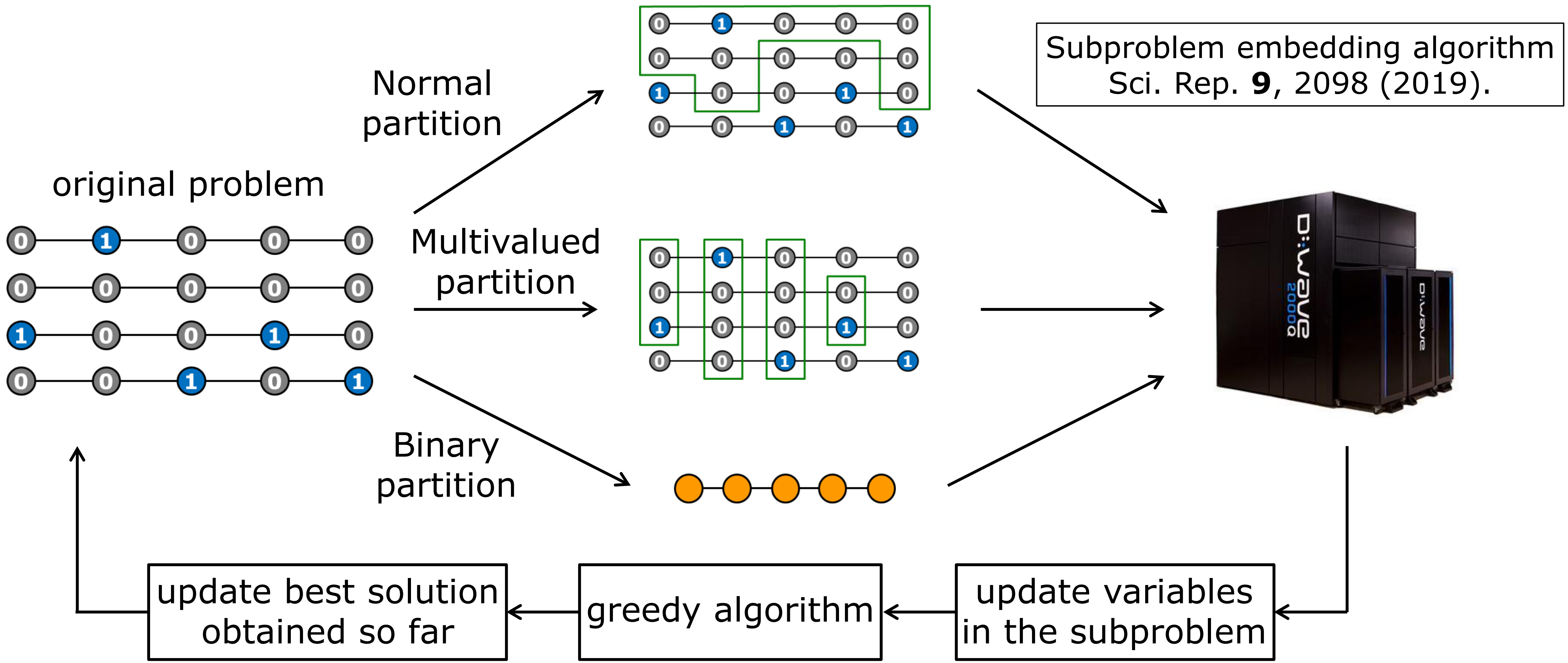


:selected component

All interactions cannot be satisfied;
non-trivial ground state

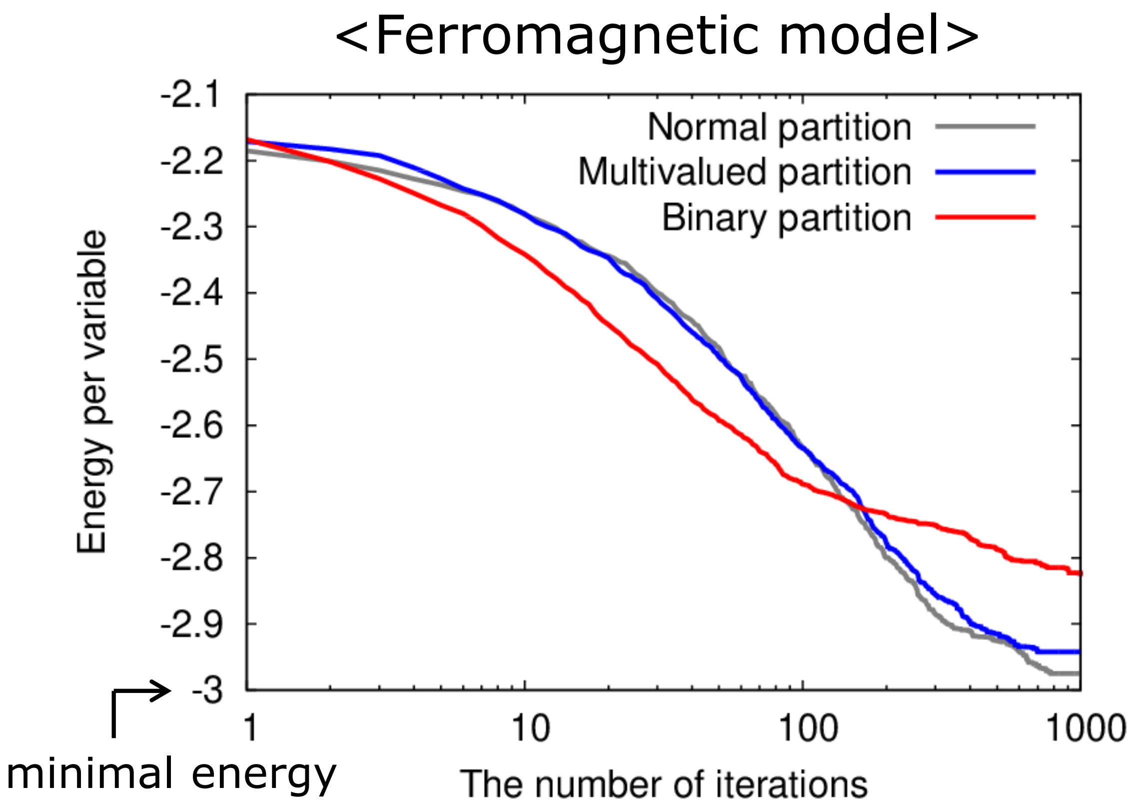
The ground states of the hard problem is non-trivial.

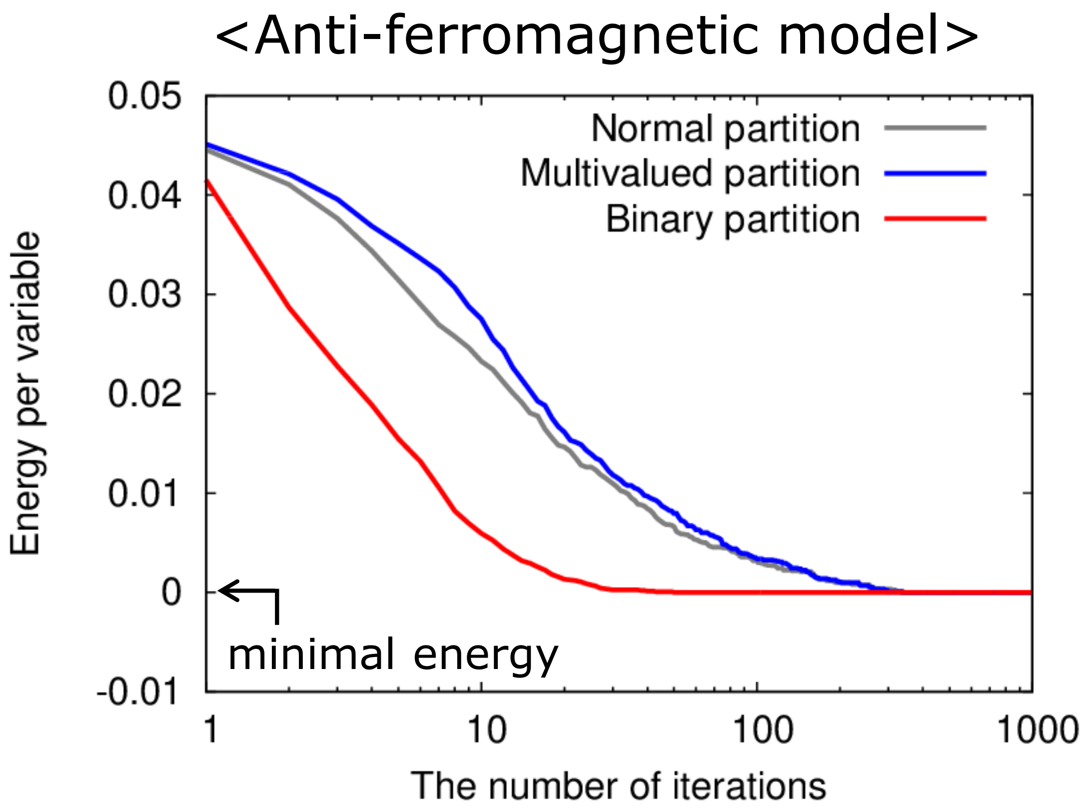
Optimization process



We compare solution accuracy for three partitioning methods.

Results for the easy problems



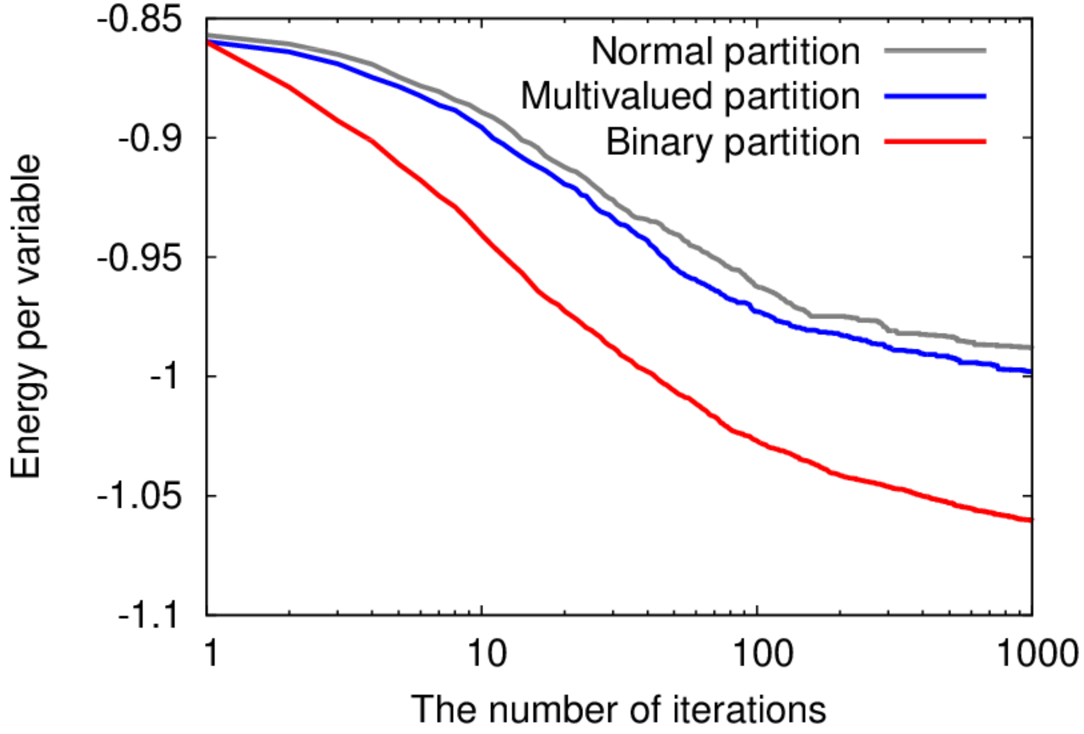


Performance of normal and multivalued partitions are almost same.

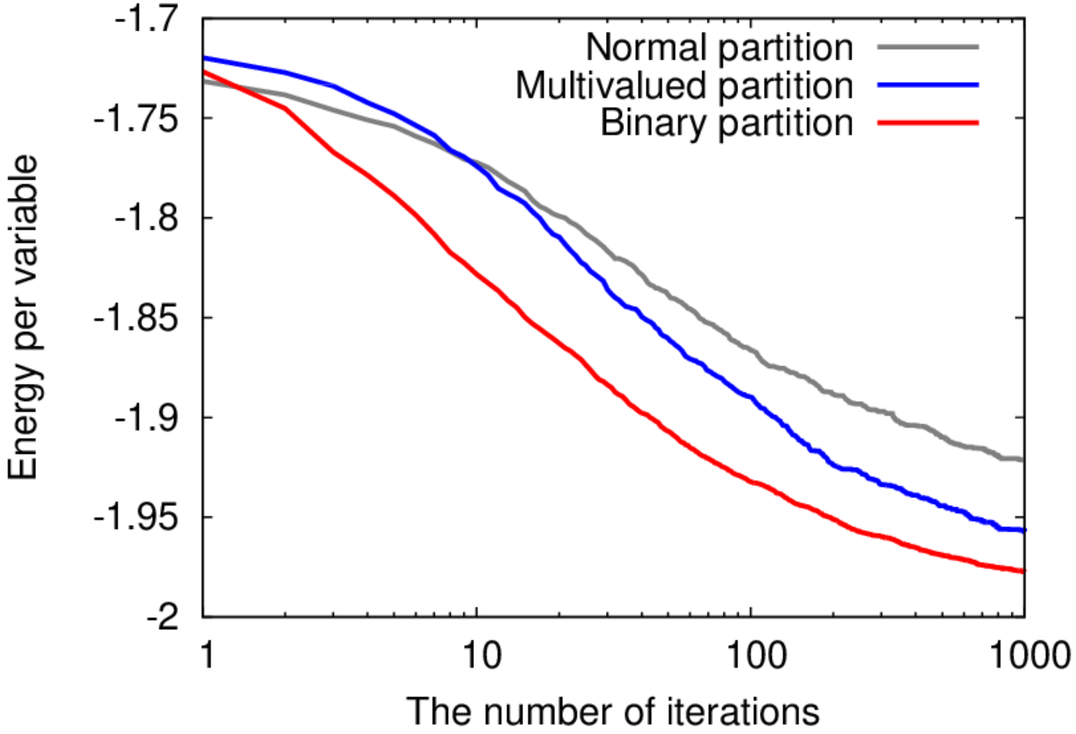
The energy obtained by binary partition differs from that of others.

Results for the hard problems

<Potts glass model>



<Potts gauge glass model>



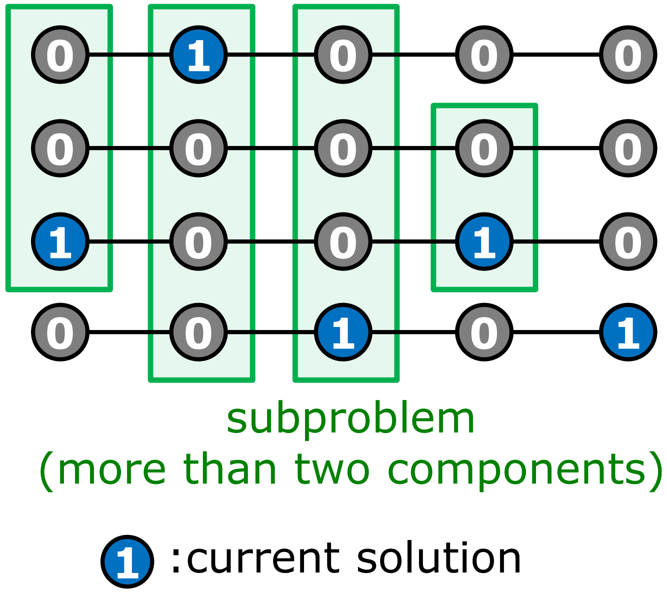
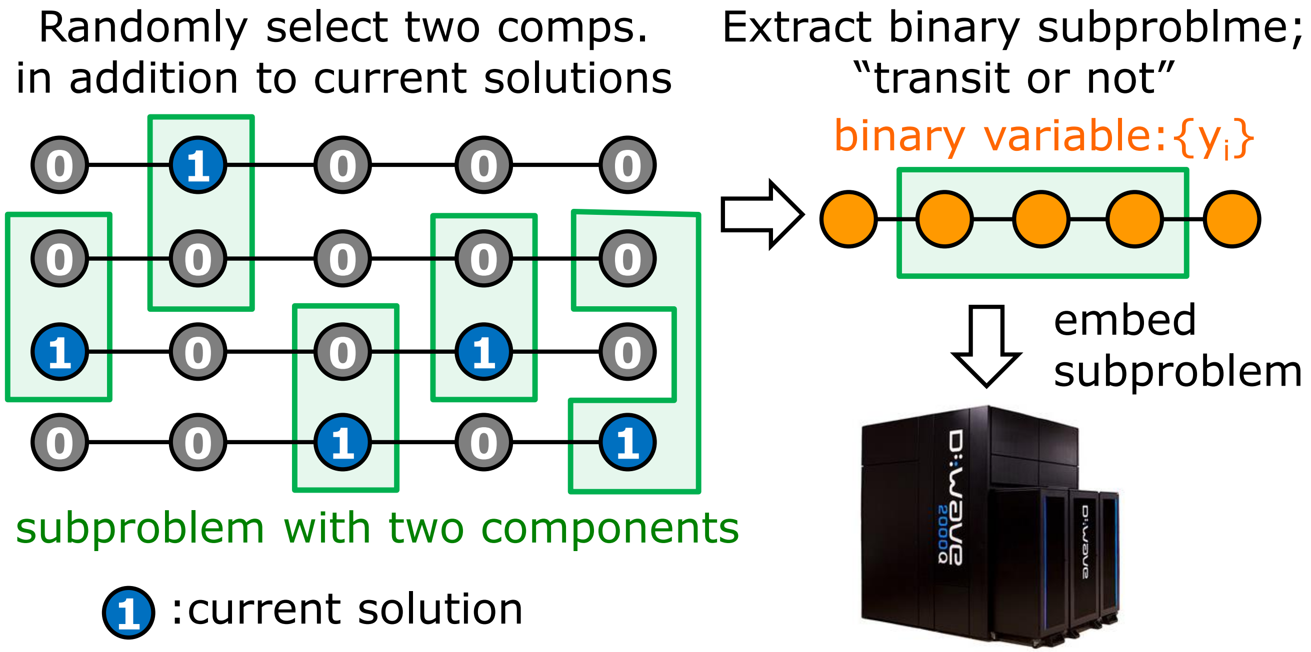
Performance of multivalued partition is better than that of normal one.
The binary partition shows the best performance for the hard problems.

4.

Discussion on the results

Proposed methods

Two methods to extract subproblems including states that satisfy the constraint.

	Multivalued partition	Binary partition
Summary	Extract subproblems that contain current solution for each variable  	Extract a binary subproblem. 
Pros	･Destinations satisfying the one-hot constraint exist for all variables.	･All states satisfy the constraint, and λ disappears. ･Large number of variables can be embedded.
Cons	･Not all states satisfy the constraint.	**･Only two components are considered at one time.**

Discussion: Binary partition for the ferromagnetic model

■Question

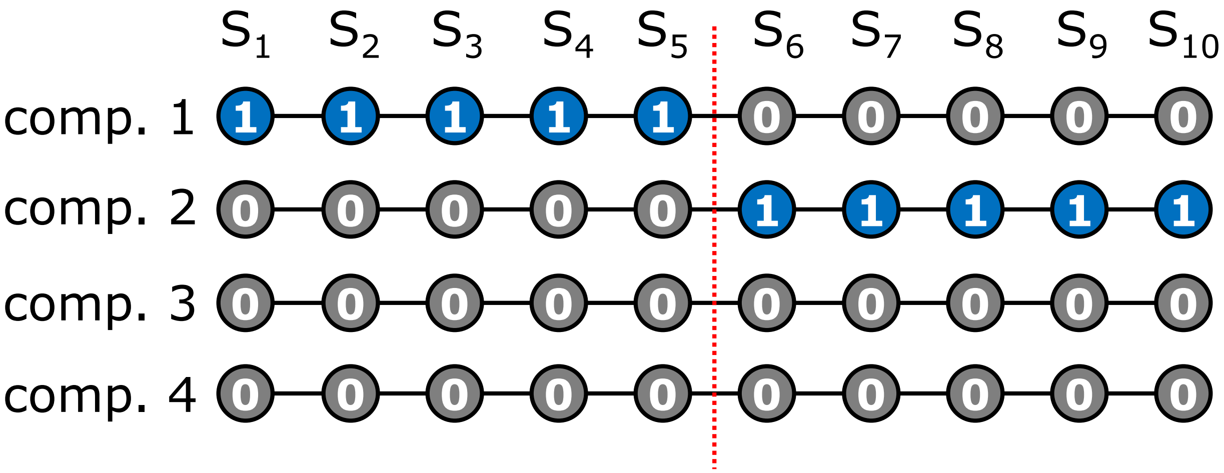
Why is the performance of the binary partition remarkably bad for the ferromagnetic model?

■Answer

Subproblems which can eliminate domain walls are rarely extracted.

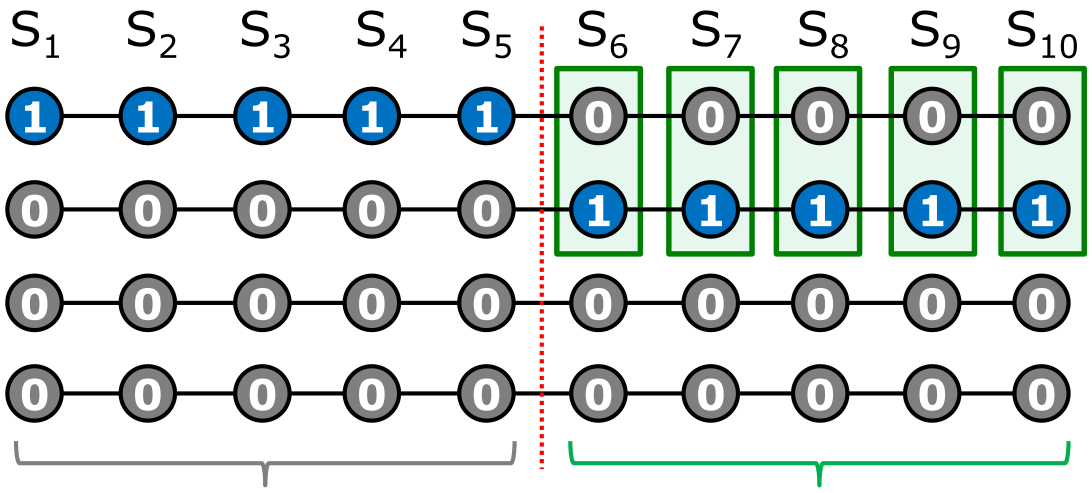
<Current solution>

1D ferromagnetic Potts model with ten variables



One of the first excited states which often appear.

<Subproblem to align all variables to comp. 1>



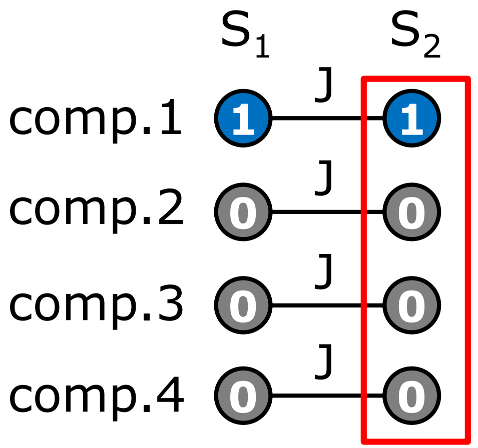
The binary partition is not suitable for the ferromagnetic model.

Discussion: Binary partition for the anti-ferromagnetic model

■Question
Why is the performance of the binary partition remarkably high for the anti-ferromagnetic model?

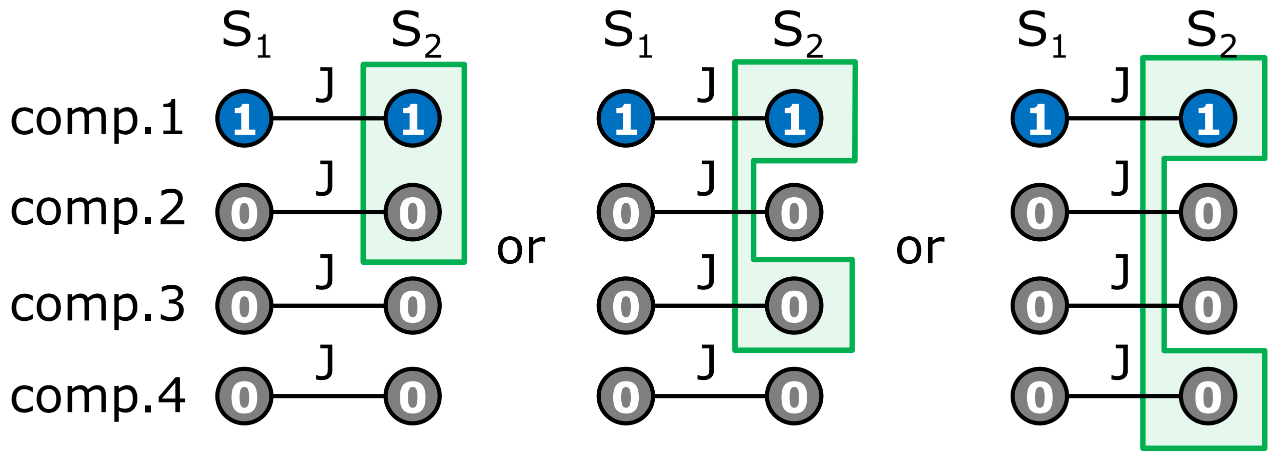
■Answer
There exist many binary subproblems that reduce the energy.

<Current solution>



Suppose that we update the variable S_2.
$S_2 \neq 1$ reduce the energy.

<Binary subproblems to improve the current solution>



All binary subproblems can improve the current solution.
⇒ Extracting only two components is sufficient.

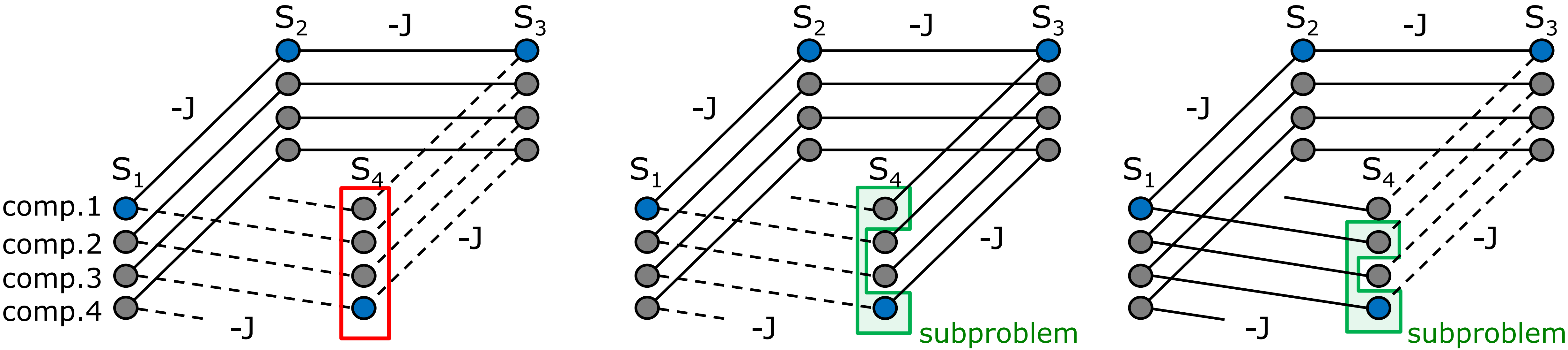
The existence of many low-energy transition destinations is essential.

Discussion: Binary partition for the hard problems

■Question
Why is the performance of the binary partition high for the hard problems?

■Answer
There exist many low-energy destinations caused by frustrations.

< Current solution > <Binary subproblems to improve the current solution>



Suppose that we update the variable S_4.
Ground states satisfy three interactions.

There are two states that satisfy three interactions
⇒ Two of three binary subproblems can reduce the energy.

The binary partition is suitable for the problems with frustrations.

Summary of this talk

■Summary

・We proposed two partitioning methods for problems with the one-hot constraint.

・The binary partition shows best performance except for the ferromagnetic model.

・The binary partition is suitable for problems with many low-energy destinations.

・The binary partition contains only constraint-satisfying states, and we do not need to adjust the parameter λ.

・We could not find problems for which the multivalued partition is suitable.

■Future work

・Construct new algorithms to efficiently optimize the ferromagnetic model using the binary partition.

DENSO

Crafting the Core

Discussion: Normal partition for the easy problems

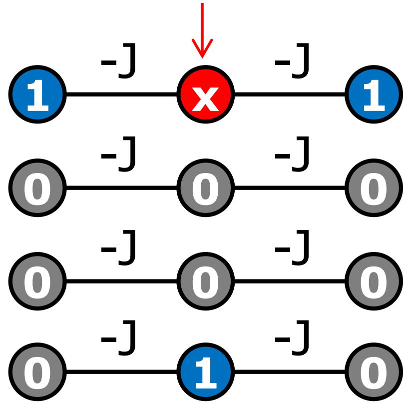
■Question
Why is the performance of the multivalued partition is not superior to the normal one?

■Answer
Transitions that violate the one-hot constraint can reduce the energy for the easy problems.

<Simple example of normal partition>

1D ferromagnetic Potts model

extract one-variable subproblem



<Energy of the subproblem>

$$E(x=1) = \underline{-2J} + \underline{\lambda}$$

interaction constraint (penalty)

$$E(x=0) = 0$$

If λ is not so large(λ < 2J)
transitions violating the constraint can reduce the energy.

Multivalued partition is not so effective in improving solutions, if there exist states that satisfy many interactions simultaneously.