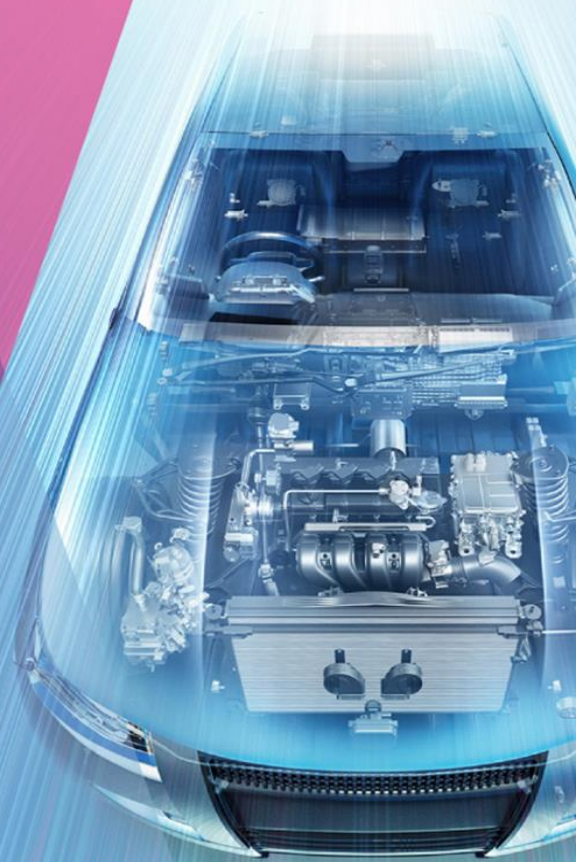
DENSO
Crafting the Core

Our Target Applications and Embedding Algorithm of subproblems



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Masayoshi Terabe[1] and Shinichiro Taguchi[1]

[1]DENSO CORPORATION
[2]Tohoku University

DENSO X QUANTUM 3

TOHOKU UNIVERSITY

Our direction

Quantum technology creates new era of Mobility, Factory and Society IoT!!

Team & projects

Mobility IoT

Multi modal service


Akira Miki


Shu Tanaka

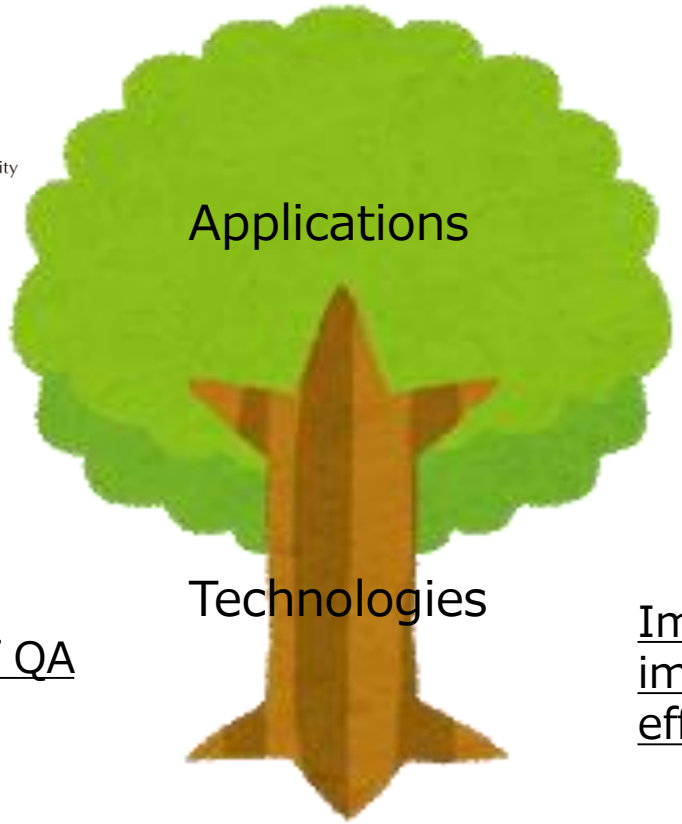
WASEDA University

Delivery service etc. in Thailand


Hirotaka Irie


Toru Awashima

TOYOTA TSUSHO



Factory IoT

Multi robot control


Akira Miki


Masayuki Ohzeki

TOHOKU UNIVERSITY

next talk by Ohzeki

Basic research of QA machine


Tadashi Kadowaki

Improvement of implementation efficiency


Shuntaro Okada

This talk

Main results of this talk

Result 1:
A fast algorithm to embed larger subproblems is proposed.

Result 2:
Better solutions are efficiently obtained by embedding larger subproblems.

Agenda

1. Improvement of solutions

2. Conventional tools

3. Proposed algorithm

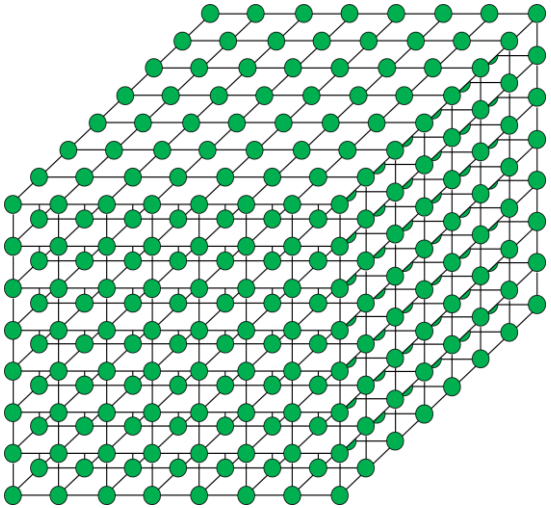
4. Performance of the proposed algorithm

1.

Improvement of solutions by embedding larger subproblems

Optimization process

cubic lattice with
1000 variables

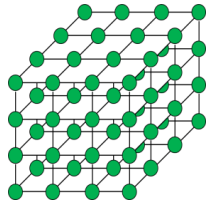


embed a subproblem
by the proposed algorithm

380 variables

optimize the subproblem
by D-Wave 2000Q_2

63 variables



extract
a subproblem

utilize the complete
graph embedding

update best obtained solution so far

greedy algorithm

update variables in the subproblem

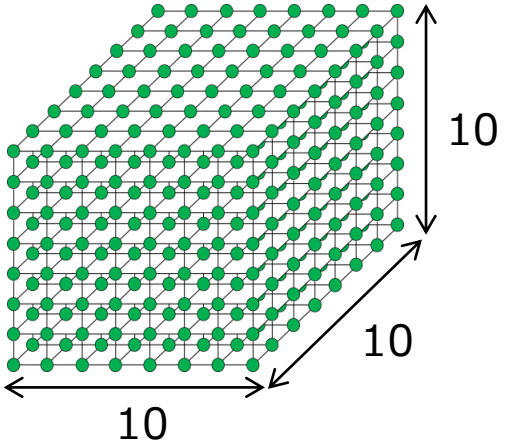
We confirmed improvement of solutions by embedding larger subproblems.

Problem settings

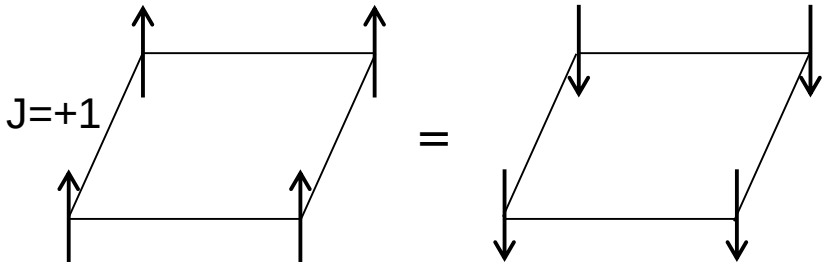
■ The cost function

$$\mathcal{H} = -\sum_{<i,j>}^{N=1000} J_{ij}\sigma_i\sigma_j$$

$$P(J_{ij}) = p_{\mathrm{F}}\delta(J_{ij} - J) + (1 - p_{\mathrm{F}})\,\delta(J_{ij} + J)$$

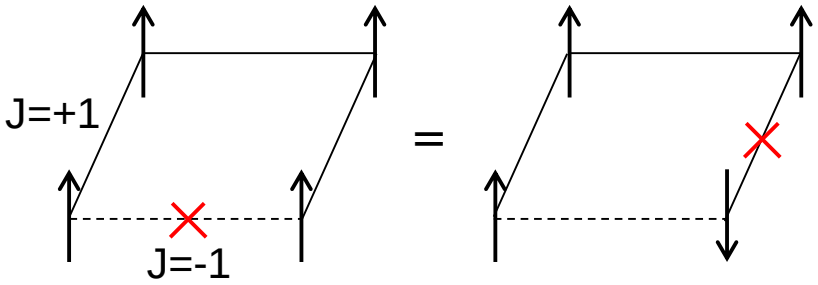


① the ferromagnetic model: pF=0.0



The trivial ground states exist.

② the spin-glass model: pF=0.5



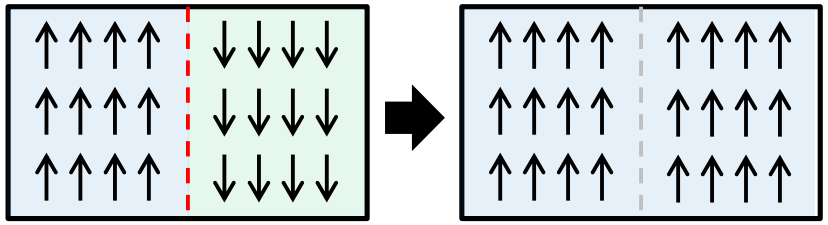
There are many frustrations.

The problem with pF=0.0 and 0.5 are solved.

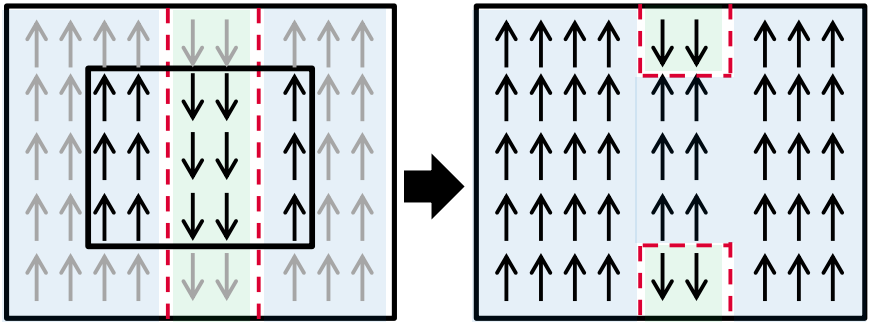
Advantages of embedding larger subproblems

■The ferromagnetic model
Eliminating domain walls is essential.

domain wall



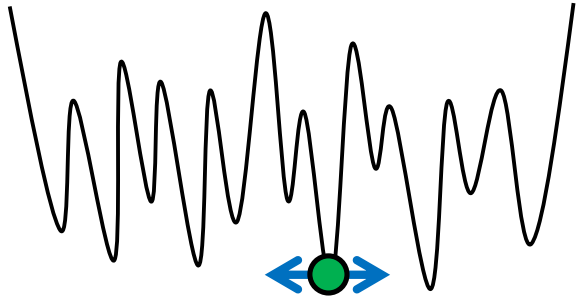
< large subproblem >



Variables in the small domain are efficiently aligned to the majority

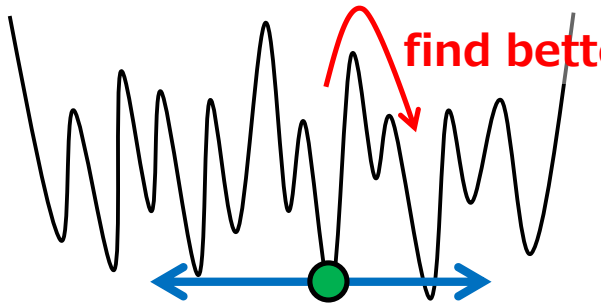
■The spin-glass model
Searching many local minima is essential.

< small subproblem >



phase space of a small subproblem

< large subproblem >

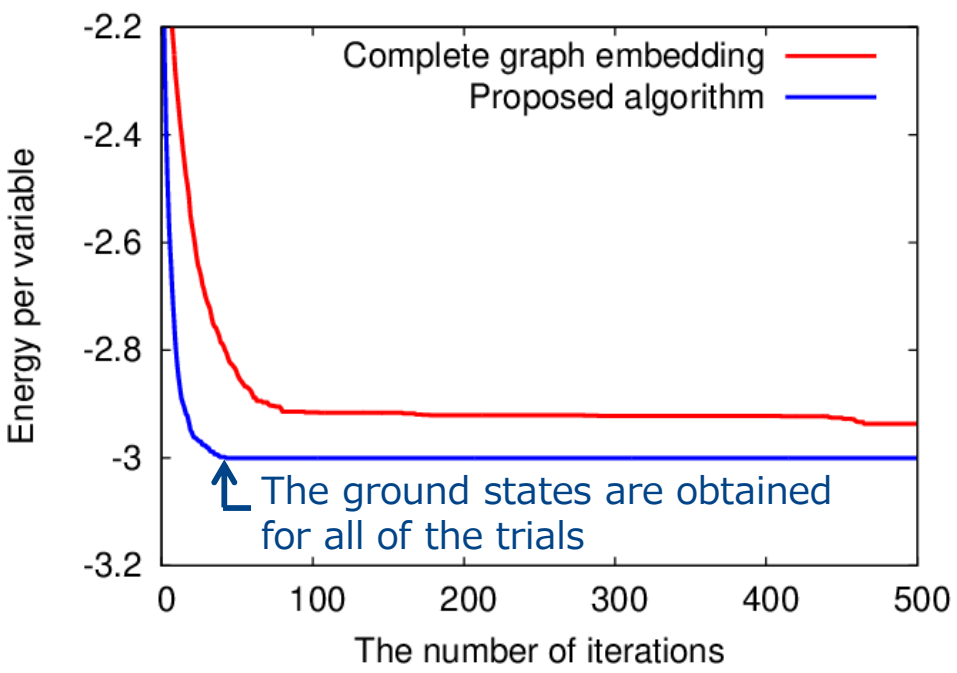


It is expected that better solutions are efficiently obtained by embedding larger subproblems.

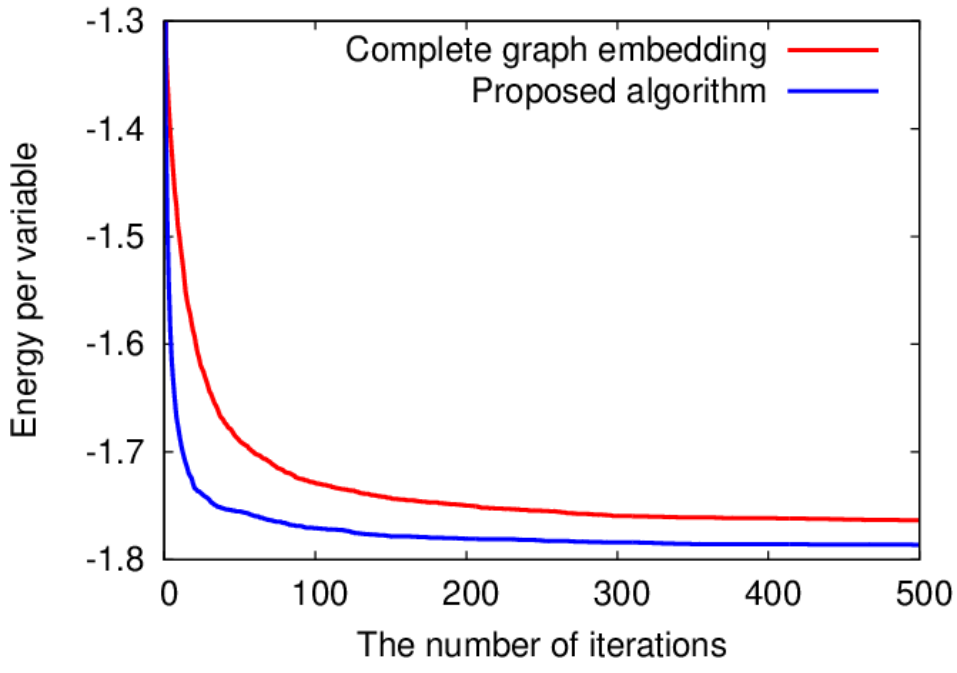
Improvement of solutions

Average energies for 32 trials are plotted in the graphs below.

< The ferromagnetic model >



< The spin-glass model >



Better solutions are obtained with the smaller number of iterations.

Domain walls are efficiently eliminated and local minima are widely searched by proposed method.

2.

Conventional tools

Limitations of the current D-Wave machine

■ Ising model of D-Wave machine

①Number of qubits

$$\mathcal{H} = \sum_{(i,j)\in Chimera}^{N_q} J_{ij}\sigma_i\sigma_j + \sum_{i=1}^{N_q} h_i\sigma_i$$

②Restricted to Chimera graph

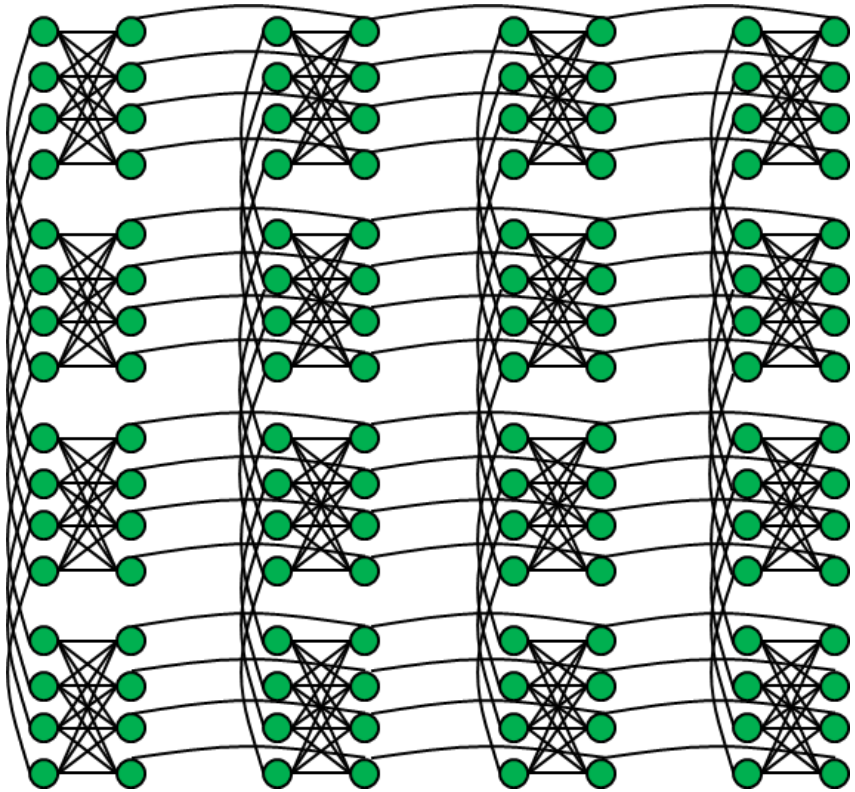
■ Practical optimization problem

①Large number of variables

$$\mathcal{H} = \sum_{i<j}^{N_p} J_{ij}x_ix_j + \sum_{i=1}^{N_p} h_ix_i$$

②Between arbitrary variables

■ Structure of Chimera graph



Partitioning and minor embedding are required to optimize practical large problems.

Conventional tools: heuristic embedding

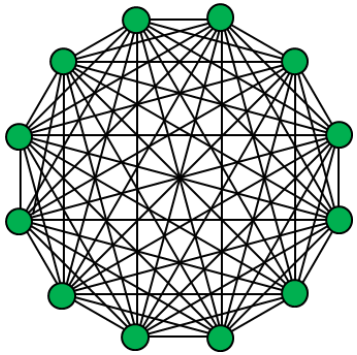
■ heuristic embedding

・Find embedding of an arbitrary problem graph to an arbitrary hardware graph

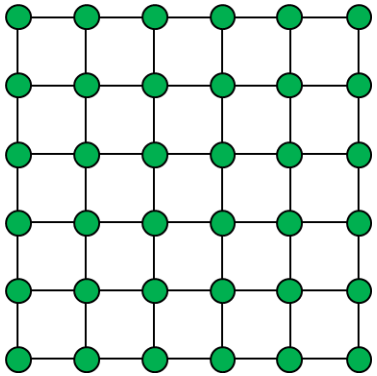
・Most versatile embedding algorithm so far

■ Problem graphs

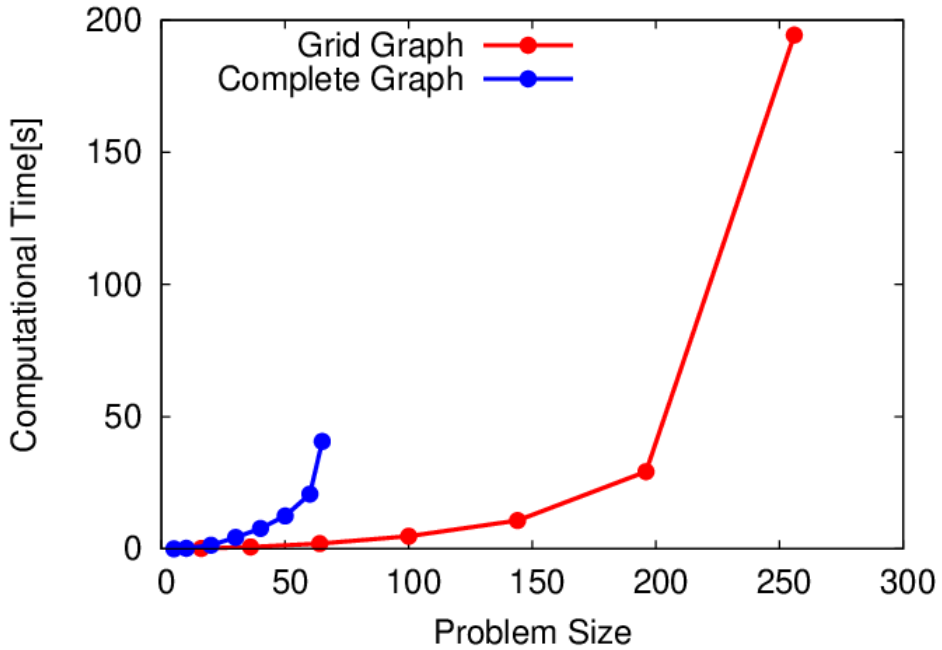
complete graph



grid graph



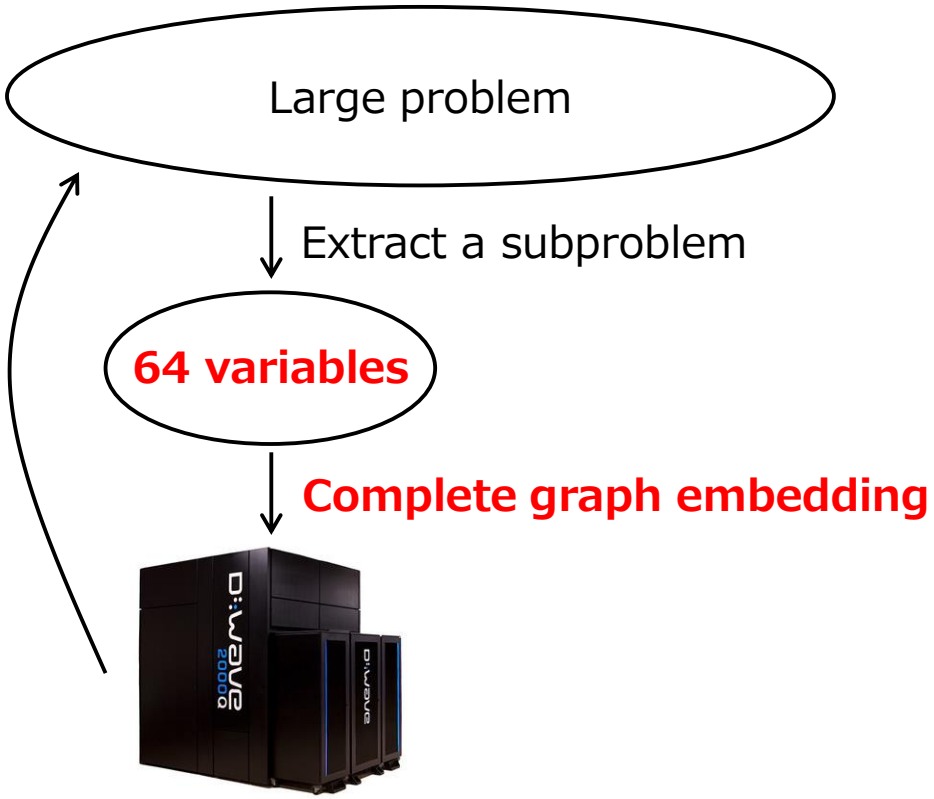
■ The Computational time



Long time is consumed for sparse problem graphs.

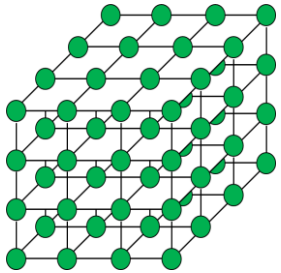
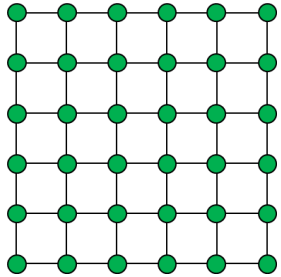
Conventional tools: qbsolv

■ Optimization process of qbsolv



The size of the subproblem is small

■ Maximum embeddable size

Cubic lattice	Grid graph
	
512	1024

The resource of D-Wave machine is not efficiently used in qbsolv.

■ Use heuristic algorithm?
The computational time is too long

Fast algorithm to embed larger subproblems is required to exploit the potential of D-Wave machine

3.

Proposed algorithm

Details of conventional heuristic algorithm 1/3

■Embedding process

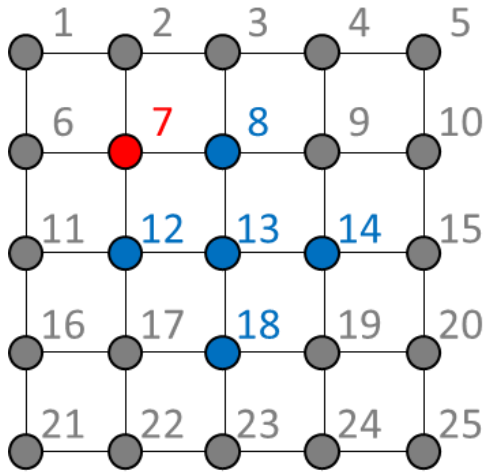
Initial stage: All of the variables are embedded once.

Last stage: The embedding obtained by the initial stage is refined.

■Initial state

One variable is selected, and embedded to Chimera graph one by one.

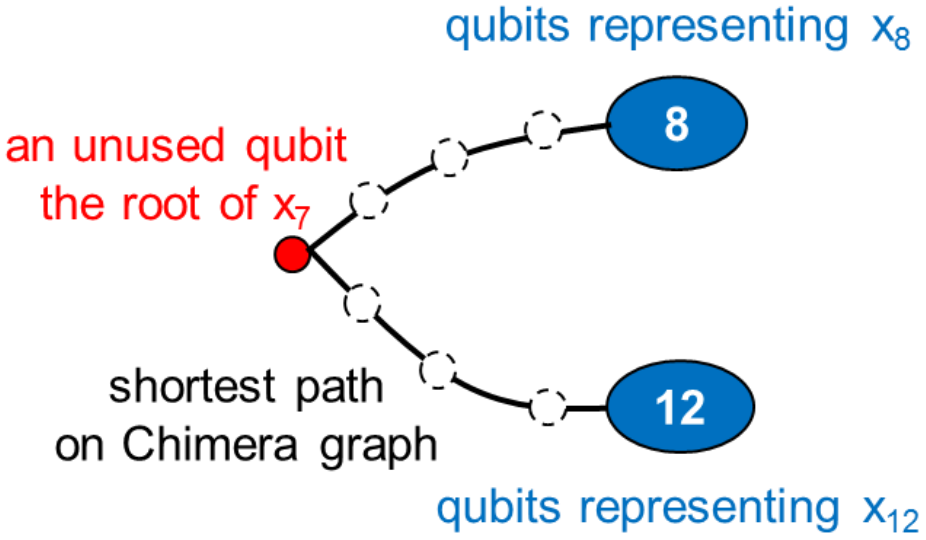
< selection of one logical variable >



●:additionally embedded

●:already embedded

< embedding of the selected variable x_7 >



All variables are embedded in the initial stage.

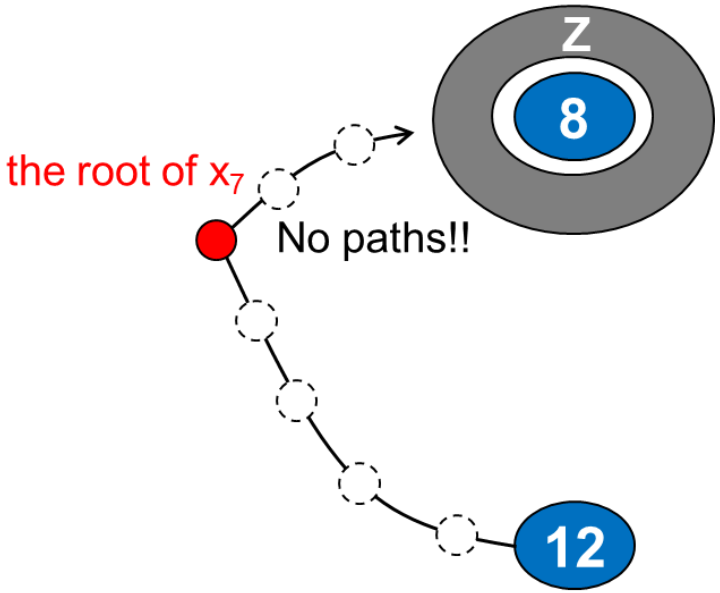
Details of conventional heuristic algorithm 2/3

■ Multiple assignments

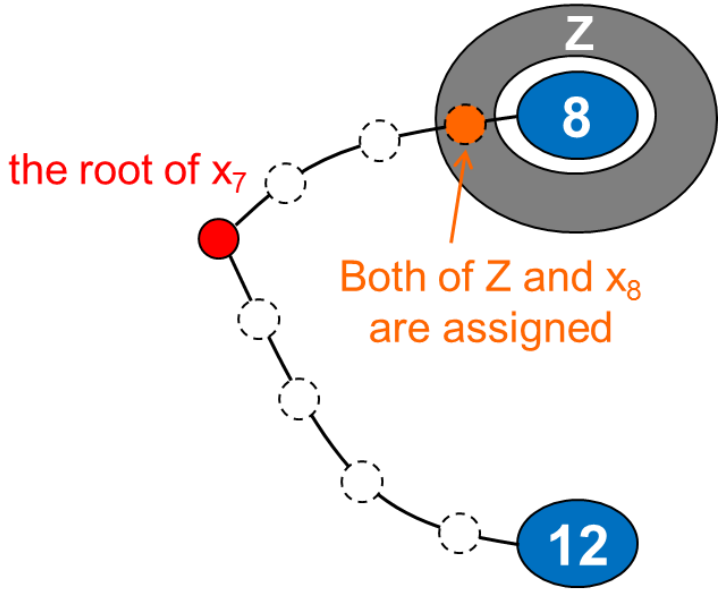
It is often the case that paths only with unused qubits do not exist

⇒ Multiple variables are assigned to one qubit in the initial stage

< No paths only with unused qubits >



< The multiple assignment >



By allowing the multiple assignments,
all of the variables are embedded in the initial stage.

Details of conventional heuristic algorithm 3/3

■ Last stage

・The embedding is refined so that one variable is assigned to one qubit.

・The refinement is conducted until the multiple assignments are all eliminated.

	The computational time
The initial stage	$T_{\mathrm{initial}} \sim O\left(E_{\mathrm{p}}\left(E_{\mathrm{q}} + N_{\mathrm{q}} \log N_{\mathrm{q}}\right)\right)$
The last stage	$T_{\mathrm{last}} \sim O\left(N_{\mathrm{q}} N_{\mathrm{p}} E_{\mathrm{p}}\left(E_{\mathrm{q}} + N_{\mathrm{q}} \log N_{\mathrm{q}}\right)\right)$

over head

common factor

Nq: the number of qubits / Np: the number of variables

The computational time in the last stage is dominant.

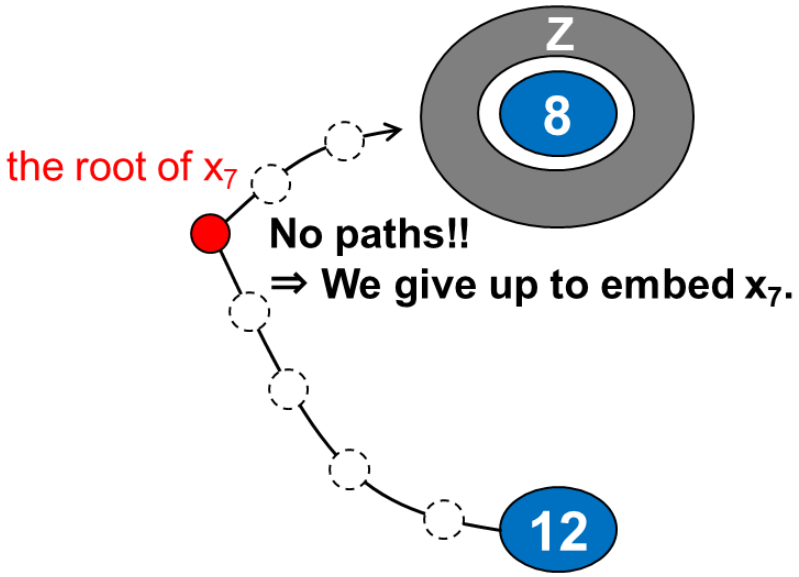
By avoiding the multiple assignments, the computational time will be drastically reduced.

Proposed algorithm

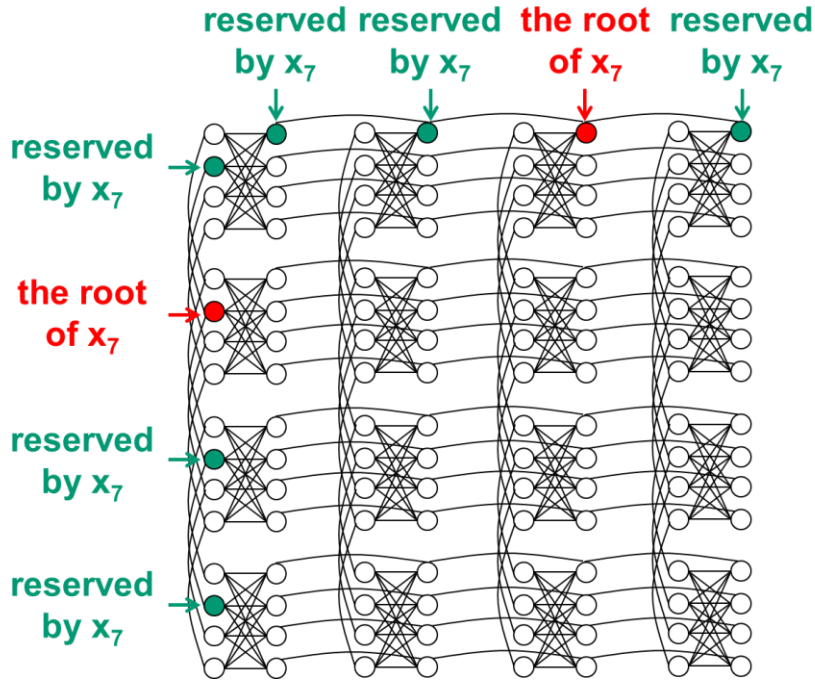
■Proposed algorithm

We focus on the embedding of “subproblem” and avoid the multiple assignments.

< Filtering of variables >
select variables embeddable without the multiple assignments



< Reservation system >
qubits are reserved related to the root to leave the space to extend a chain



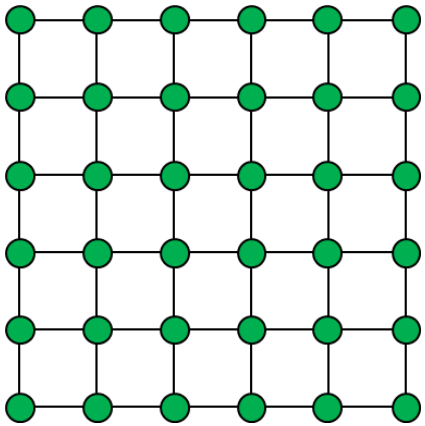
The filtering of variables and reservation system are introduced to avoid the multiple assignments.

4.

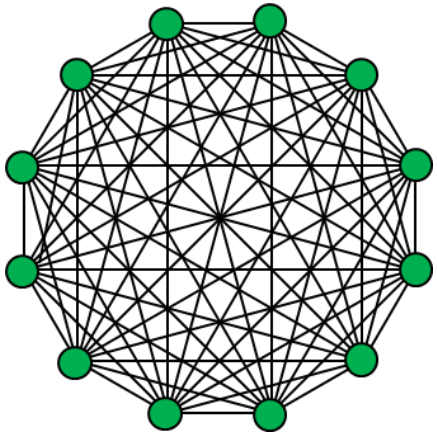
Performance of the proposed algorithm

Dependence on the number of qubits

grid graph with
300 x 300 variables

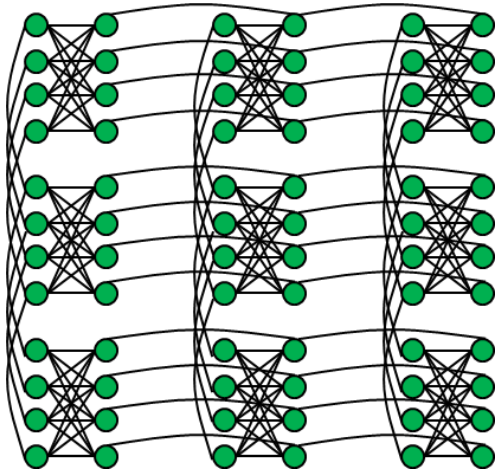


complete graph with
1000 variables



embedding of
subproblems

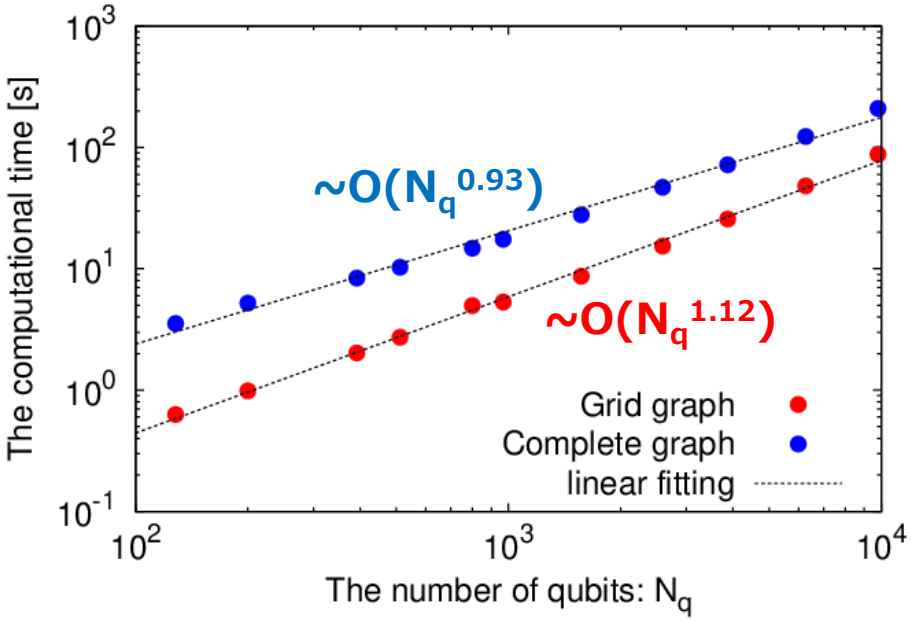
Chimera graph with
10^2~10^4 qubits



We evaluate the dependence of the computational time and the size of subproblem on the number of qubits.

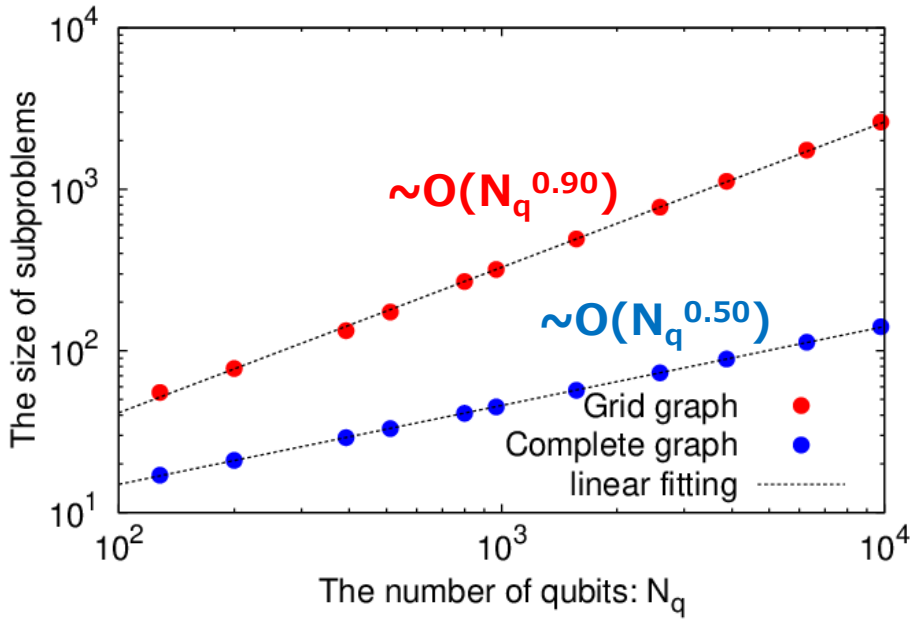
Computational time and the size of subproblems

< The computational time >



The computational time increases slowly with respect to Nq

< The size of subproblems >

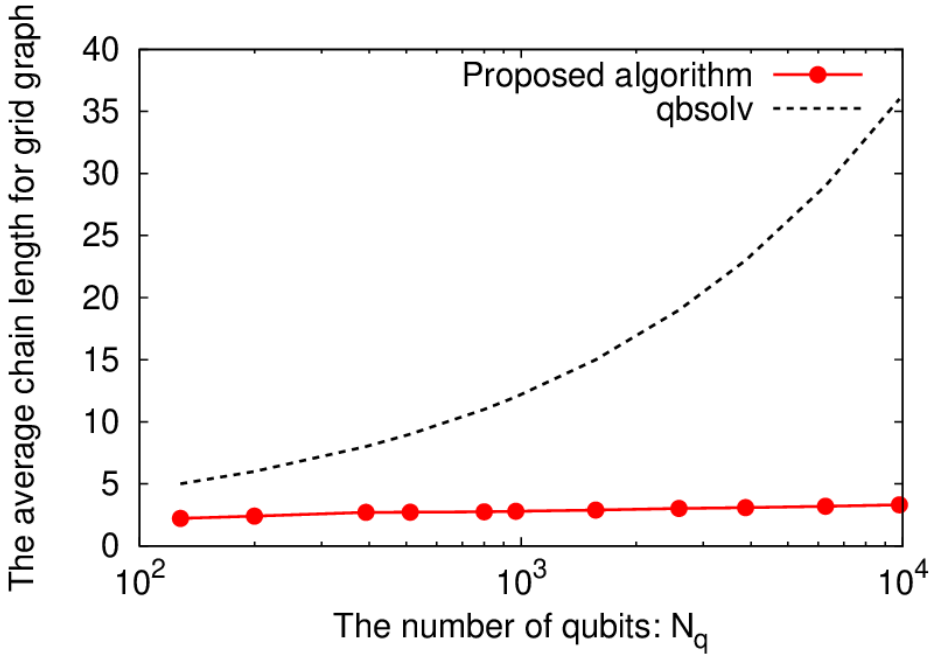


Larger subproblems are embedded for grid graph than that of the complete graph

Large subproblems can be embedded depending on the connectivity with the shorter computational time.

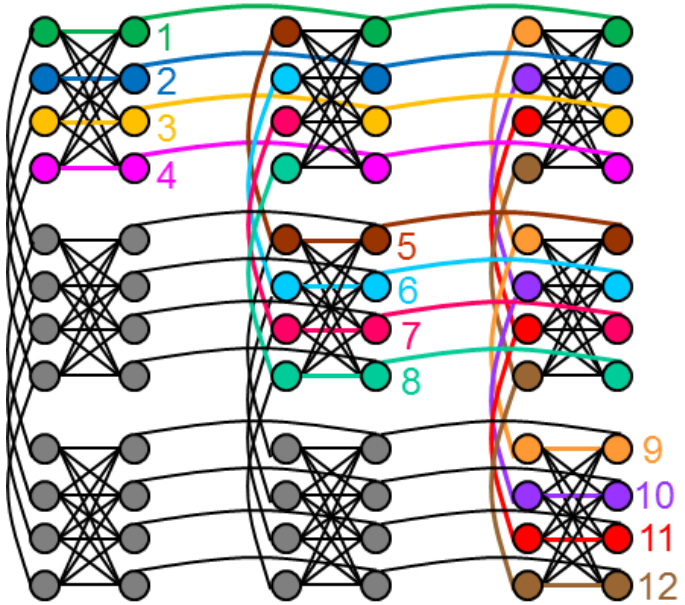
Average chain length for the grid graph

< Comparison of chain length >



The chain length becomes shorter than the complete graph embedding used in qbsolv.

< Complete graph embedding >



The chain length is uniform for all of variables.

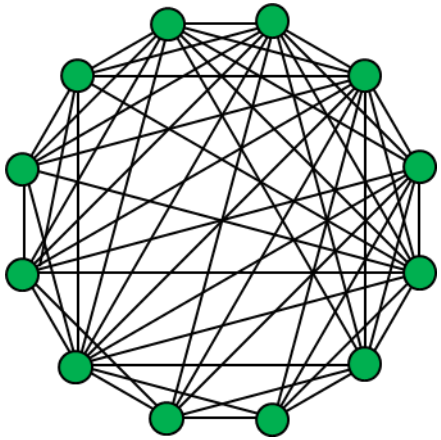
It is expected that the shorter chain length will also contribute to the improvements of solutions.

Subproblem embedding of Erdős–Rényi model

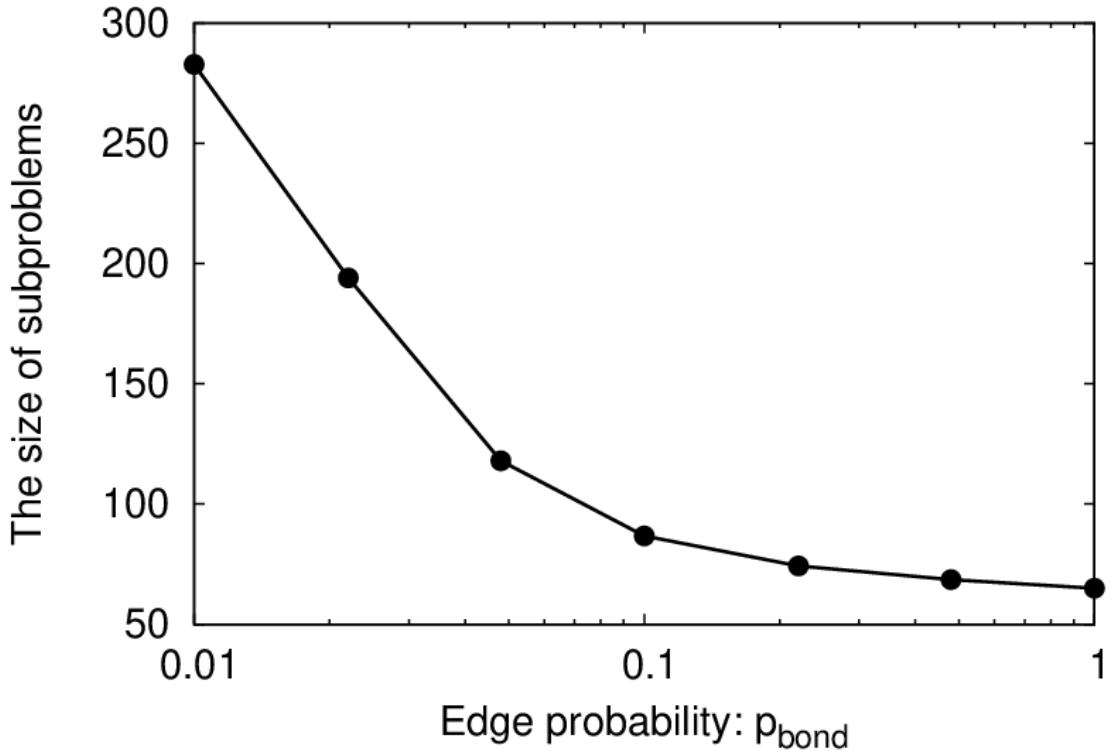
< problem graph >

$$\mathcal{H} = -\sum_{i>j}^{N=1000} J_{ij}\sigma_i\sigma_j$$

$$P\left(J_{ij} \neq 0\right) = p_{\mathrm{bond}}$$



< The size of subproblems >



The proposed algorithm can embed subproblems of Erdős–Rényi random graph.

Summary and future work

■ Summary

・We proposed a fast algorithm to embed larger subproblems.

・In the proposed algorithm, the multiple assignments of variables are avoided to reduce the computational time.

・For the ferromagnetic model and the spin-glass model on the cubic lattice with 1000 variables, better solutions are obtained with the smaller number of iterations by embedding larger subproblems.

・Domain walls are efficiently eliminated and local minima are widely searched by embedding larger subproblems.

■ Future work

・We confirm the validity of the proposed algorithm for various problems.

・The computational time reduction such as parallelization and so on.

Follow us!!

・Source Code of the proposed algorithm
Coming soon in Git Hub
(https://github.com/denso-quantum/sub_embed)

・Homepage
https://www.denso.com/global/en/innovation/technology/quantum/

・Twitter
@densoquantum

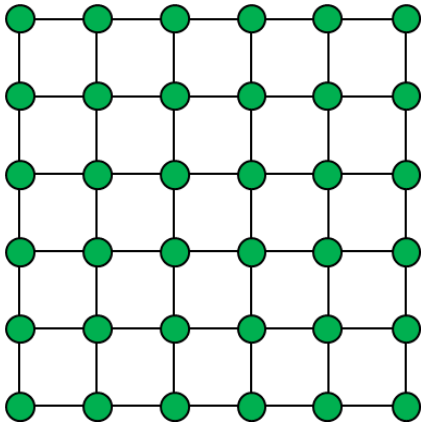
DENSO

Crafting the Core

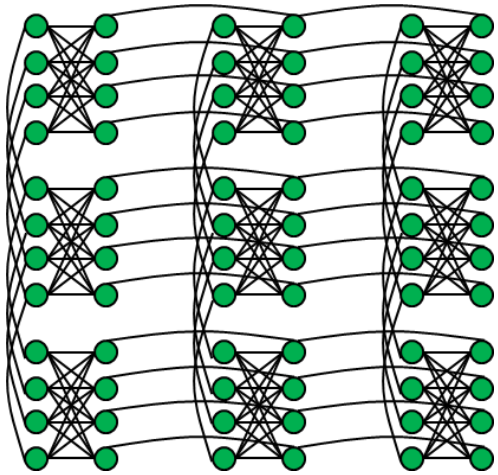
Example of an extracted subproblem

< extracted subproblem >

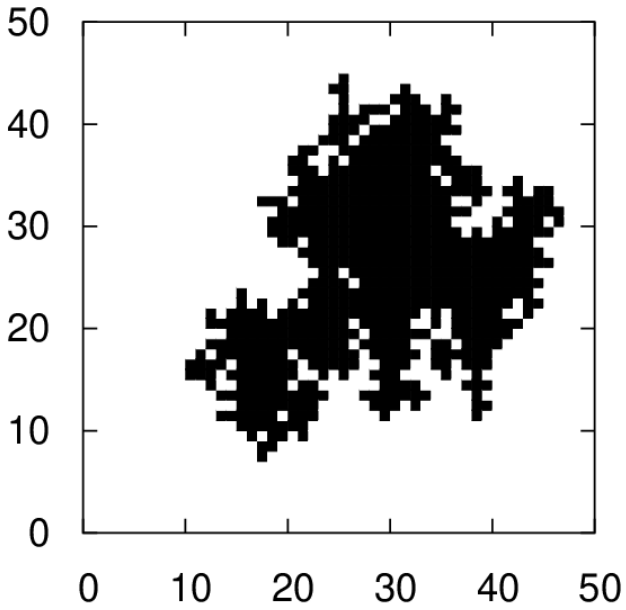
Grid graph with
50 x 50 variables



Chimera graph
with 2048 qubits

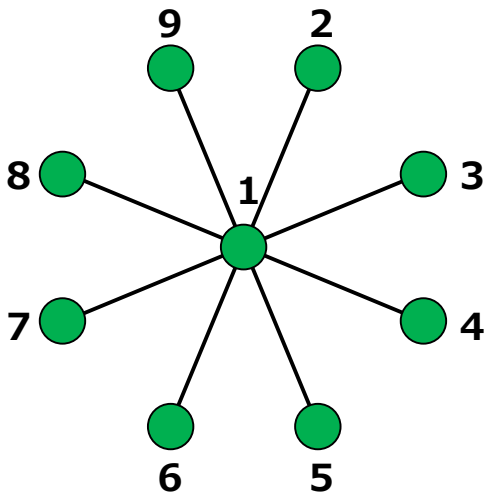


< extracted subproblem >

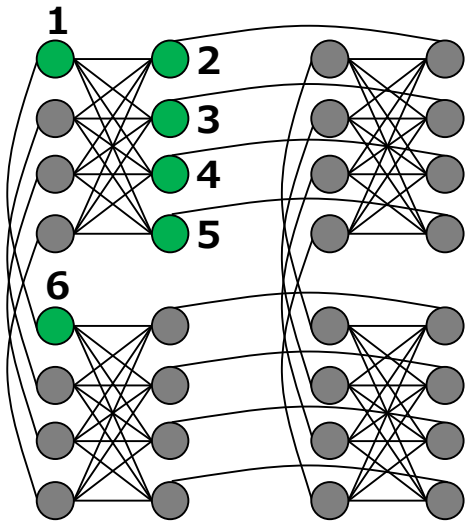


Without the reservation system

< problem graph >

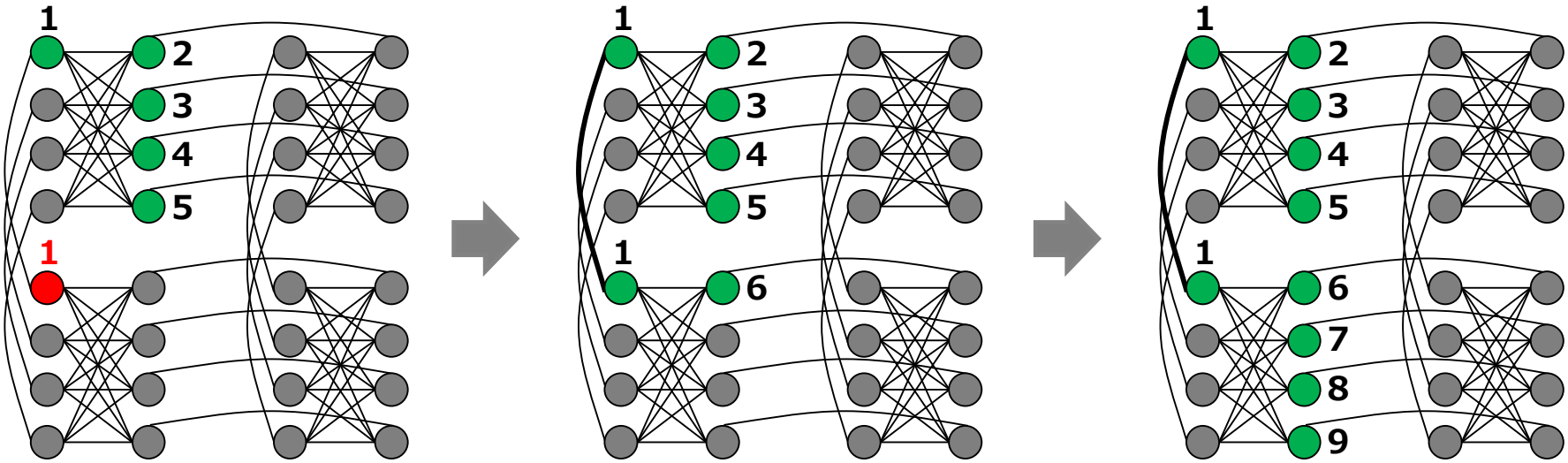


< without reservation system >



The variables embeddable without the multiple assignments will disappear soon.

With the reservation system



The dense problem graphs can be embedded by introducing the reservation system.